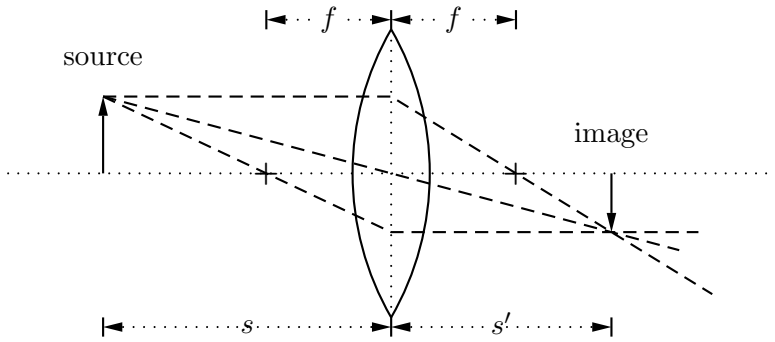
Physics 117 Lab Manual



Amherst College

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General Instructions

The laboratory sessions of Physics 117 are designed to help you become more familiar with fundamental physical concepts by actually carrying out quantitative measurements of physical phenomena. The labs are designed to help you develop several basic skills and several "higher-level" skills. The **basic** skills include the following:

1. Being able to relate abstract concepts to observable quantities. For example, knowing how one determines the electrical resistance of a device from easily measured quantities. This skill includes the ability to estimate and measure important physical quantities at various levels of precision.
2. Knowing and applying some generally useful measurement techniques for improving the reliability and precision of measurements, such as using repeated measurements.
3. Being able to estimate the experimental uncertainties in quantities obtained from measurements.

The **higher-level** skills include the following:

1. Planning and preparing for measurements.
2. Executing and checking measurements intelligently.
3. Analyzing the results of measurements both numerically and, wherever applicable, graphically. This skill includes assessing experimental uncertainties and deviations from expected results.
4. Being able to describe, talk about, and write about physical measurements.

The laboratory work can be divided into three parts: **preparation, execution,** and **analysis.** The preparation, of course, must be done **before you come to your laboratory session.** The execution and analysis (for the most part) will be done during the three-hour laboratory sessions. Some suggestions for performing these three parts successfully are given below. Please also refer to Appendix A, **Keeping a Laboratory Notebook**, since good note-taking will be essential in all phases of the lab.

Preparation

1. Read the laboratory instruction carefully. Make sure that you understand what the ultimate **goal** of the experiment is.
2. Review relevant concepts in the text and in your lecture notes.
3. Outline the measurements to be made.
4. Understand how one goes from the measured quantities to the desired results.
5. Organize tables for recording data and the equations needed to relate measured quantities to the desired results.

Execution

1. One of the most important elements of executing the experiment will be keeping a step-by-step record of what you've done, how you've done it, and in what order. You will use an inexpensive, permanently **bound notebook** provided to you for recording your laboratory data, your analysis of the data, and the conclusions you draw from the measured results along with any other relevant comments.

   The notebook is an informal record of your work, but it must be sufficiently neat and well-organized so that both you and the instructor can understand exactly what you have done. It is also advantageous for your own professional development that you form the habit of keeping notes on your experimental work — notes of sufficient clarity that you can understand them at a later time. Developing good lab notebook technique requires consistent effort and discipline, skills that will be of great value in any professional career. If you become a research scientist, you will often (while writing reports or planning a new experiment) find yourself referring back to work you have done months or even years before; it is essential that your notes be sufficiently complete and unambiguous that you can understand exactly what you did then. In keeping a laboratory notebook, it is better to err on the side of verbosity and redundancy than to leave out possibly important details.

   NEVER ERASE data or calculations from your notebook. If you have a good reason to suspect some data (for example, you forgot to turn on a power supply in the system) or a calculation (you entered the wrong numbers in your calculator), simply draw a line through the data or calculation you wish to ignore and write a comment in the margin. It is surprising how often "wrong" data sets turn out to be useful after all. In fact, **use a pen to record all of your data** so that you will not fall prey to the temptation to erase.

   As mentioned above, Appendix A gives instructions on how to keep a good lab notebook. You will be expected to adhere to these guidelines throughout the semester. In fact, we feel that keeping a good laboratory notebook is so important that **we have decided to base part of your laboratory grade (equivalent to one formal lab**

report) on the quality of your lab notes. Your notebook will be evaluated at the end of the semester.

Note: You should **have your lab notebook initialed** by one of the instructors before you leave each lab session.

2. Naturally, you will need to pay attention to your data-taking technique. Throughout the semester, you will be learning how to use various types of measurement equipment, sometimes crude and sometimes sophisticated. In all cases, the quality of your data will depend on your understanding when and how to use the equipment most effectively. It is always more important to put care and thought into the setup for a measurement than it is to attain a (sometimes deceptively) high level of accuracy from a meter. Of course you will want to optimize the accuracy, but only when you're sure you're making the **right** measurement. For example, if you build a circuit incorrectly, it doesn't matter how many digits you get out of a voltage reading. If the circuit is wired wrong, the results will be wrong, too. So take care to think first, and always critically assess your measurements as you go along to see if they make sense. In fact, this is really a preliminary part of the analysis process.

Analysis

1. In addition to the ongoing analysis you conduct during data-taking, you will be expected to perform a more thorough analysis for each experiment. Most importantly, you will be asked to obtain meaningful physical results from the measurements. Often, though not always, this will be done in the context of a graphical analysis. That is, usually you will use the "method of straight-line graphing" (see Appendix C) that you have come to know and love (from Physics 16/23, or equivalent) to create plots that theoretically "should" be linear. From the slopes of these graphs, you will often be able to determine a result that is effectively an average measurement from all of your data. Of secondary (though not insignificant) importance is the need for you to specify some limits of accuracy about your results. This phase of the operation is often mistakenly referred to as "Error Analysis." In fact, the expression for determining the range of uncertainty in a particular measurement is, appropriately, "Uncertainty Analysis." Below are some guidelines to help with this process.

2. Uncertainties:
   The stated results of any measurement is incomplete unless accompanied by the uncertainty in the measured quantity. By the uncertainty, we mean simply: How much greater, or smaller, than the stated value could the measured quantity have been before you could tell the difference with your measuring instruments? If, for instance, you measure the distance between two marks as 2.85 cm, and judge that you can estimate halves of mm (the finest gradations on your meter stick), you should report your results as 2.85 ± 0.05 cm. More details on uncertainties are given in Appendix F.

   An important (if not *the* most important) part of the analysis of an experiment is an assessment of the agreement between the actual results of the experiment and

the expected results of the experiment. The expected results might be based on theoretical calculations or the results obtained by other experiments. If you have correctly determined the experimental uncertainty for your results, you should expect your results to agree with the theoretical or previously determined results within the combined uncertainties. If your results do not agree with the expected results, you must determine why. Several common possibilities are the following:

(a) You underestimated the experimental uncertainties;
(b) There is an undetected "systematic error" in your measurement;
(c) The theoretical calculation is in error;
(d) The previous measurements are in error; or
(e) Some combination of the above.

Sometimes these deviations are "real" and indicate that something interesting has been discovered. In most cases (unfortunately) the explanation of the deviation is rather mundane (but nevertheless important). Remember that small deviations from expected results have led to several Nobel prizes.

Lab Reports

You will prepare a report for each laboratory session. We will have two types: (1) short (informal) reports with an exit interview conducted by one of the laboratory instructors; and (2) longer, written (formal) reports. Both types depend on your having kept a careful record of your work in the lab notebook.

Informal reports will, in general, focus on your in-class record of the experiment during the lab time along with your answers to the questions posed in the handouts for each lab. These short reports need not describe the entire experiment; however, they should be complete and self-contained and distinct from any pre-lab lecture notes which are also written in your lab notebook. Your notebooks will be collected at the end of informal labs and graded.

Formal reports will be required for three of the labs (see the Laboratory Syllabus). For formal reports, you are to prepare a somewhat longer, written account of your experimental work. These reports should include a complete description of the experiment and its results. They should be typed (you will likely prefer to use a word processor) on separate sheets of paper (not in your lab notebook) and are to be turned in within one week after the end of lab. You should pay special attention to the clarity and conciseness of your writing. In fact, if we find that your report would benefit from rewriting, we may ask you to submit a revised version of the report before a grade is assigned to the report. Guidelines for preparation of formal lab reports are included on a subsequent sheet in this manual.

Please hand in your lab reports on or before the due date; as with the other written exercises in the class, **late work will not be accepted**.

Grading

You must complete all of the labs to pass Physics 117. If you have to miss a lab because of illness, family difficulties, or other legitimate reasons, please let your instructor know in advance (whenever possible) so we can arrange for a make-up time.

You will receive a grade for each of the formal lab reports. These grades, along with an evaluation of your lab notebook (which will be weighted like one formal lab report) and an overall evaluation of your performance during the labs will constitute your lab contribution for the course grade.

Intellectual Responsibility

Discussion and cooperation between lab partners is strongly encouraged and, indeed, often essential during the lab sessions. **However, each student must keep a separate record of the data and must do all calculations independently and must write an independent lab report.** It is strongly advised that students do not communicate with each other, in person or electronically, once the writing process has begun. Specific questions concerning the writing of reports should be directed to the instructor or teaching fellow. In addition, laboratory partners are expected to share equally in the collection of data. The sharing of drafts of reports, use of any data or calculations other than one's own, or the modeling of discussion or analysis after that found in another student's report, is considered a violation of the statement of intellectual responsibility.

We wish to emphasize that intellectual responsibility in lab work extends beyond simply not copying someone else's work to include the notion of scientific integrity, i.e., "respect for the data." By this we mean you should not alter, "fudge," or make up data just to have your results agree with some predetermined notions. Analysis of the data may occasionally cause you to question the validity of those data. It is always best to admit that your results do not turn out the way you had anticipated and to try to understand what went wrong. You should NEVER ERASE data that appear to be wrong. It is perfectly legitimate to state that you are going to ignore some data in your final analysis if you have a justifiable reason to suspect a particular observation or calculation.

Experiment 1

Kirchhoff's Rules

For the first half of the semester you will study various topics of electromagnetism. One of the most successful applications of electromagnetism is the development of electric circuits. You will construct simple circuits and study their electrical properties. To have some initial understanding of how circuits work you will employ two simple rules of electric circuits, Kirchhoff's Rules. At the heart of these rules are laws of conservation; specifically the law of conservation of energy which you learn about in P116 or P123, and the conservation of electric charge. Before we can go further we will need to introduce two more electrical quantities; namely electrical current and electrical potential difference or voltage.

1.1 Background

In this lab you will be causing electric charge to flow through a variety of materials. This flow of charge will not be covered in lecture for several more weeks. In lab, therefore, we will be taking a less formal, more phenomenological point of view.

The motion of charge is called an electric current in analogy to the flow of fluid. Consider the flow of a fluid, like water, through a pipe (Fig. 1.1).

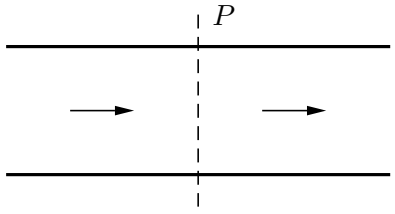

Figure 1.1. Flow of fluid across a plane P.

The current through this pipe is characterized by the amount of fluid, measured by volume flowing past a plane P per unit time. Similarly, if charge is flowing down a wire the electrical current is characterized by the amount of charge flowing through P per unit time. The symbol for electric current is I and its units are amperes (amp,A):

$$\boxed{1 \text{ ampere} = 1 \text{ C/s} = 1 \text{ A}}$$

To get water to flow through a pipe there must be a pressure difference between one end and the other. The analogous concept with electrical circuits is electrical potential difference. The symbol for electric potential difference is V, and its units are volts (V):

$$\boxed{1 \text{ volt} = 1 \text{ J/C} = 1 \text{ V}}$$

Hooking both ends of a hose up to the inlet and outlet of a water pump will cause water to flow through the hose. A battery plays a similar role for electric circuits. Attaching one end of a wire to the positive end of a battery and the other to the negative end will cause charge to flow through the wire. (Please take our word for this and refrain from discharging the batteries in this way.) Batteries are special kinds of electrical pumps because they pretty much always maintain the same potential difference between the positive and negative ends no matter what the current.

The ratio between the potential difference between say the ends of a wire and the electric current flowing through the wire is called the resistance, R, of the wire

$$R = \frac{\Delta V}{I} \tag{1.1}$$

The units of resistance are ohms (Ω):

$$\boxed{1 \text{ ohm} = 1 \text{ V/A} = 1\ \Omega}$$

Because electrical circuits can get enormously complicated, an elaborate system of symbols has been developed to help us draw the circuits we study. A battery, for instance, is represented by the symbols in Fig. 1.2. The longer vertical line indicates the position of the positive end of the battery.

A resistor is represented by the drawing in Fig. 1.3.

A circuit which in "reality" looks something like that shown in Fig. 1.4 is represented symbolically like that shown in Fig. 1.5. Current, by definition, flows out the positive end of the battery as shown.

Figure 1.2. A simple cell (left) and battery (right).

Figure 1.3. A resistor.

To measure electric current we use a device called an ammeter, which is represented symbolically in Fig. 1.6. Current must flow *through* an ammeter (Fig. 1.7). The ammeter will display the current that flows into its "red" lead and out of its "black" lead. If the measured current is negative, then the current is actually flowing into the black lead and out of the red lead. Sometimes these meter leads are referred to as "positive" (red) and "negative" (black) — for now, let's agree to eschew this convention, which often brings with it unnecessary confusion.

To measure electric potential differences we use a two-terminal device called a voltmeter, represented symbolically in Fig. 1.8. Voltmeters measure the potential *difference* between points a and b, i.e., if the potential is V_a at point a and V_b at point b, the voltmeter will display $V_a - V_b$. As with the ammeter, there is a "red" (a) and a "black" lead (b) — the price you pay for confusing these two leads is an extra minus sign in the voltage displayed by the meter. Please note that you should **NOT** set up your circuit to require current to flow through a voltmeter (Fig. 1.9).

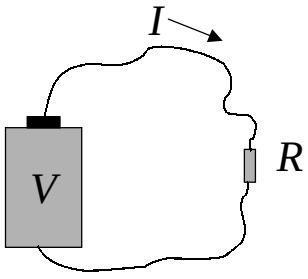

Figure 1.4. A "real" circuit with battery and resistor.

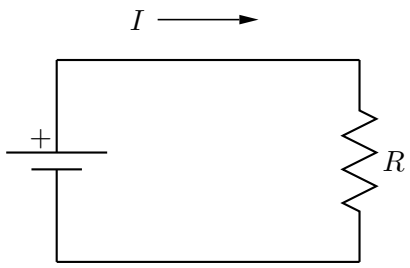

Figure 1.5. A simple circuit diagram.

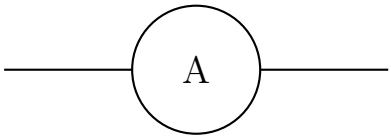

Figure 1.6. The ammeter.

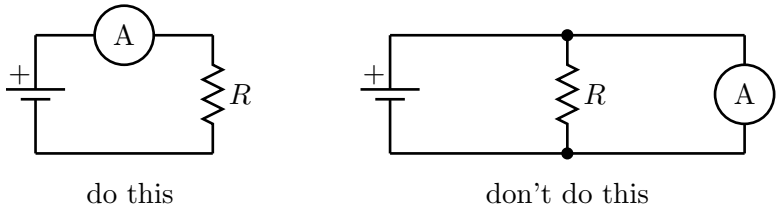

Figure 1.7. Use the setup on the left, not the right, for an ammeter.

1.2 Digital Multimeter

Today we will use a digital multimeter to make measurements of current and voltage. As the name implies a digital multimeter is a multi function device. It can be set to make measurements like a voltmeter, ammeter and more. You will typically have two multimeters on hand so you can make simultaneous measurements of current and voltage. Fig. 1.10 is a picture of the multimeter we will use in lab.

1.3 The Experiment

In this experiment you will construct a simple circuit and utilize an ammeter and voltmeter to make measurements of the current and voltage respectively. Begin by constructing a circuit using your breadboard power supply (Fig 1.11), ammeter and 470 Ω resistor provided (you can identify the resistor by the *yellow-purple-brown* bands around it). Remember, in order to measure current the ammeter must be a part of the circuit. Use Fig 1.7 to help

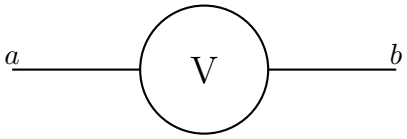

Figure 1.8. The voltmeter.

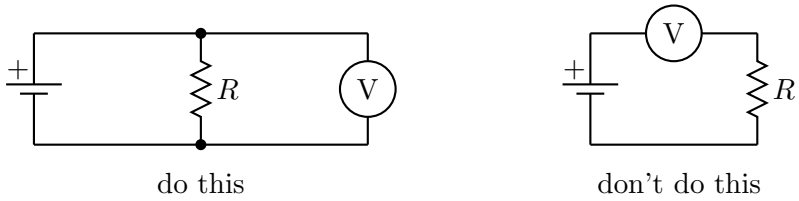

Figure 1.9. Hook up a voltmeter as shown on the left, not as shown on the right.

you construct the circuit.

- Use the screwdriver to turn the variable resistors or *pots* all the way *clockwise*. Be careful not to twist too much.
- Record the current as measured by the ammeter. The uncertainty in the measurement is ± 1 unit of the last digit.
- Use the analog voltmeter to measure the voltage across the 470 Ω resistor (Fig 1.9). Estimate the uncertainty.
- Now use Eq. (1.1) to calculate the value of the resistance of the resistor. Does the calculated value agree with the given value? What is the uncertainty in the calculated value? Use Eq. (F.38) to propagate the uncertainty.
- Now calculate the *power* dissipated by the resistor. In electrical terms the power is defined:

  $$P = I\Delta V \tag{1.2}$$

  Power is measured in units of J/sec or watts. Using the units of voltage and current, can you show the product of voltage and current gives power in watts?
- In calculating resistance and power, you are multiplying or dividing two measured quantities, each with its own uncertainty. Use Eq. (F.37) to propagate the uncertainty in the power.
- Now replace the analog voltmeter with the second digital multimeter in the voltage mode and measure the voltage across the resistor again. What is the uncertainty in the new measurement of voltage? Use the new measured voltage and calculate the resistance and power.

Figure 1.10. Triplett 1101-B digital multimeter

1.3.1 Series and Parallel Circuits

Now let's see what happens when more than one device is in the circuit. Take the red device and connect it in *series* with the 470 Ω resistor. A series circuit is one in which there is only one path for the current to flow. If this is true, then right away we know that the current through one device is the same as the current through other device.

- Measure the voltage across each device using the digital voltmeter. What is the sum of the voltages? What is the uncertainty of the sum of voltages?
- Now measure the voltage across the terminals of your power supply. How does this compare to the sum of the voltages across the two devices?

This exercise illustrates one of two principles that underlie much of our understanding of circuitry. The principle is call Kirchhoff's Voltage Rule. The voltage rule states:

The algebraic sum of voltages around a closed circuit loop is zero.

$$\sum \Delta V_i = 0 \tag{1.3}$$

How do we interpret our measurements in the context of Kirchhoff's Voltage Rules? The power supply is providing energy to establish a current in the circuit (remember the units of voltage is Joules/Coulomb). So between the negative and positive terminals of the power supply, the potential increases. As the current flows the energy is consumed to move the current through each device (i.e. resistor), so there is a potential *drop* across each device.

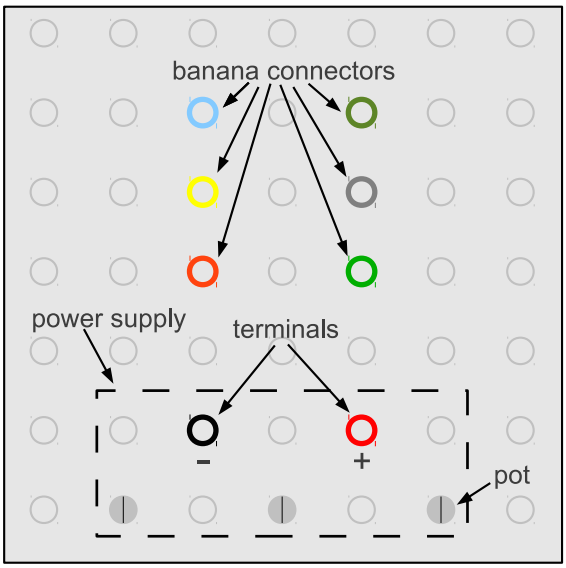

Figure 1.11. The breadboard power supply.

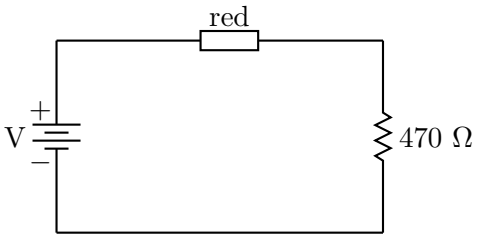

Figure 1.12. Series circuit.

Therefore the total potential provided by the battery is matched by the total potential that drops across the devices. For your circuit, that means:

$$V_{supply} - V_{470\Omega} - V_{red} = 0 \tag{1.4}$$

Now construct a circuit in which the 470 Ω resistor and the red device are in parallel with each other. Two devices are in parallel with each other when they share two junctions. In this case the *voltage* across each device is the same.

- Measure the current flowing out of the battery, i.e. before the first junction.
- Measure the current through each device. How does the sum of the individual currents compare to the current before the first junction?

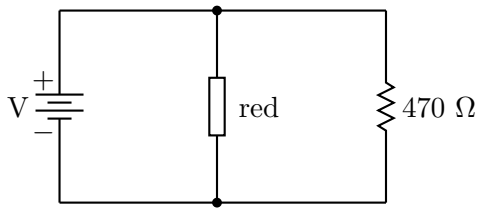

Figure 1.13. Parallel circuit.

This illustrates a second principle called Kirchhoff's Current Rule:

The sum of currents flowing into a junction is equal to the sum of currents flowing out of a junction.

$$\sum I_i = \sum I_o \tag{1.5}$$

Let's consider the case of the 470 Ω resistor and the red device:

$$I_{in} \quad = \quad I_{470\Omega} + I_{red} \tag{1.6}$$

1.4 Supplement: Power Supply Circuit Diagram

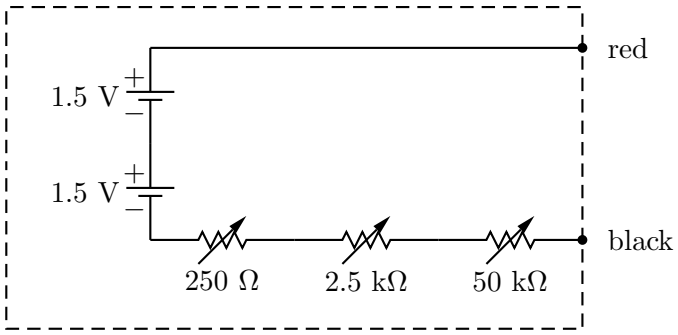

Figure 1.14. Power supply circuit diagram.

This symbol of a squiggly line with an arrow through it denotes a variable resistor, whose value can be varied by turning a knob or, in our case, a screw.

Often we will "abbreviate" the circuit diagram above like this.

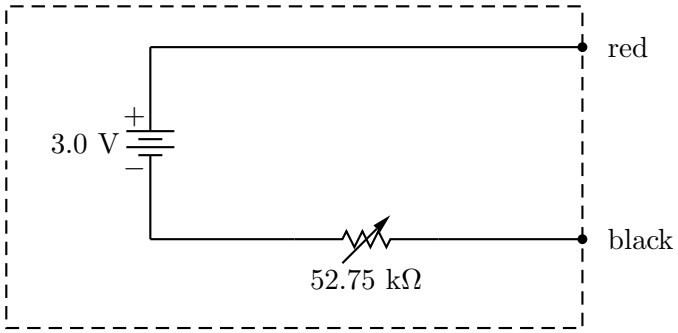

Figure 1.15. Abbreviated power supply circuit diagram.

NOTE: The board on which the power supply is mounted also holds four independent auxiliary binding posts, which may be convenient for mounting resistors, etc.

Experiment 2

Ohm's Law (Formal)

2.1 Ohm's Law

Electronic devices are often classified in terms of various classes. Devices for which R is a constant are said to be **ohmic**.

Ohm's Law:
$\Delta V = IR$
$R =$ constant

Devices that obey Ohm's Law are called resistors.

2.1.1 Equipment

You will be given the following:

1. A "power supply" consisting of two nominal 1.5 V cells in series with a set of variable resistors; a diagram of this circuit is given in the appendix to these notes. (The word "nominal" is a code word in physics that means "don't trust that this is actually so until you have measured it," i.e., *caveat emptor*.)
2. Two digital multi-meters: one used as a voltmeter and the other as an ammeter.
3. Three unknown devices: one red, one white, and one blue.

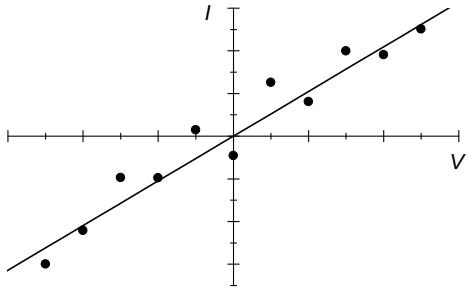

Figure 2.1. How your I–V graph might look.

2.1.2 The Experiment

Construct a circuit which will enable you to determine whether your three unknown devices have current-voltage characteristics described by Ohm's Law. Use your circuit to obtain 5 to 10 (I,V) values for each device. Your laboratory notebook should contain a current-voltage graph for each of the three unknown devices. **IMPORTANT:** Be certain to check whether or not these (I,V) values depend upon the direction in which the current flows through the unknown — you'll have to figure out how to reverse the direction of the current at some point so that you can get data for current going both directions. **BEWARE:** reversing the meter leads does NOT reverse the direction of current through the device you are testing, but it does multiply your subsequent measurements by the factor -1.

Your graphs should have the origin somewhere in the middle of the sheet, so that an I–V graph would look something like that shown in Fig. 2.1:

Which of these devices obey Ohm's Law? Determine the resistances of those that do. What can you say about the electrical properties of those unknowns (if any) that do not obey Ohm's Law?

What limits the precision of your measurement? How could you get around this limitation?

2.1.3 Role of Ammeter and Voltmeter

The resistance of your ammeter is nominally very small and that of your voltmeter is approximately 10 MΩ. Does the fact that these resistances do not have the ideal values of zero (for the ammeter) and infinity (for the voltmeter) affect your interpretation of the results of Sec. 2.1.2? Be prepared to explain qualitatively how your results would be affected by these particular meter resistances.

2.1.4 Resistors in Series and Parallel (Optional)

Suitably alter your circuit to test the hypothesis that resistors in series add according to the equation

$$R = R_1 + R_2 \tag{2.1}$$

and that resistors in parallel add according to

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{2.2}$$

Again, five to ten data points are all that are required for each case. (**Note:** you should still be measuring current through the resistors in the same way you did in Sec. 2.1.2.)

2.2 The Laboratory Report

This experiment will be written as your first formal lab report. Please follow the guidelines for writing a formal lab report in appendix E. Your report should include:

- diagrams of the circuits used,
- all data in neat and organized tables,
- graphs of the data. The graphs should be titled and axes should be labeled.
- answers to the questions raised,
- and a conclusion of the results and findings.

Experiment 3

Introduction to the Oscilloscope

The oscilloscope is one of the most powerful and versatile devices in science and technical fields, from physics to medicine (e.g. heart monitors). In today's experiment, you will become familiar with some of the common operations of an oscilloscope. Ask your instructors to tell you more details about the history and inner workings of your scope.

3.1 Comments

- It is impossible to damage one of these scopes by twiddling the knobs. So do not be inhibited—try things out and see what happens. (It is possible, however to damage them by dropping them on the floor, hitting them with a hammer or spilling water on them. Please resist any temptation to do these things.)
- These scopes may seem at first to have a bewildering array of knobs and switches. That is the price we pay for versatility. We will suggest some initial settings. As you learn to know and love these scopes, you will become more adventurous. If you get too adventurous too quickly and lose the signal altogether, first try (by thinking about the functions of the various controls) to get it back; if you fail, go back to the suggested initial settings. If that fails, an instructor may be able to help.

3.2 Oscilloscope

The oscilloscopes we will use in this class are Tektronix 2001c, which can measure signals up to about 50 MHz on two input channels. They are general-purpose digital oscilloscopes, of a kind that has become nearly ubiquitous in research laboratories around the country. Unlike the digital multimeter, you must plug these oscilloscopes in, although there are now

battery-powered versions that are liberated from the grid.

Figure 3.1. The Tektronix 2001c. (Figure from the Tektronix user manual.)

3.2.1 Display

Once you have plugged in the oscilloscope, you can turn it on by pressing the power button on the top of the device. After a few moments of self-testing the main display will show a grid and, possibly, yellow and/or blue lines similar to Fig. 3.2.

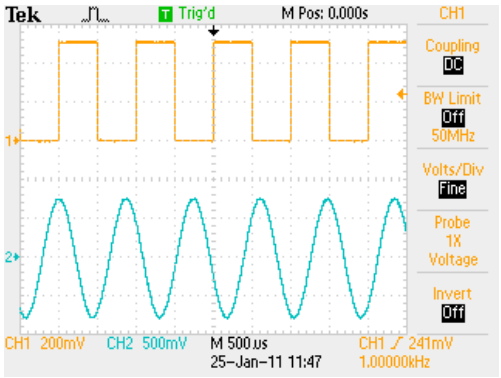

Figure 3.2. A typical oscilloscope display, with channel 1 in yellow and channel 2 in blue. You will have only the square wave trace at this point. The softmenu selections for channel 1 appear to the right of the voltage graph.

The main display graphs the voltage appearing on one (or both) of its input channels as a function of time. This is a major improvement over the voltage averaging provided by the multimeter, but with this great power comes great responsibility, as attested by the many controls on the front panel. In this lab we will spend time on the most essential oscilloscope controls.

3.2.2 Inputs

There are three inputs to the oscilloscope (see Fig. 3.3). The two signal inputs are the first two on the left (marked 1 and 2); we may talk about the third input (marked Ext Trig) later. These inputs are so-called BNC[1] connectors. There are special BNC cables that can be plugged into these connectors; you should try this out now, to get the sense of how these connectors work. Note that they push on and then turn to make the solid connection. The voltage difference on a BNC cable is measured between the central pin and the outer shield. There are also adapters that convert from BNC to banana, in which case there is a red connector and a black connector. These are very similar to the connectors on the handheld multimeter, with one important exception that we will discuss in a little while.

Figure 3.3. The three inputs to the oscilloscope. (Figure from the Tektronix user manual.)

3.3 Procedure

1. **Initial Settings.**

   (a) Push the 1 button to activate the CH 1 menu. If you press the button again the trace will disappear. You can make it reappear if you press it once more. You should now see some menu items on the right of the screen, each associated with a "soft key" button just to its right.

   (b) Using the appropriate "soft key" to the right of the screen, set the CH 1 Coupling to DC.

   (c) Adjust the vertical Scale knob for CH 1 to 1 VOLT/DIV. The value of the scale appears in the bottom left corner of the screen (Fig 3.2).

   (d) Push the 2 button to activate the CH 2 menu. Set up this channel to also have 1 VOLT/DIV and DC Coupling.

[1]The letters BNC stand for Bayonet Neill Concelman, after the style of the connector and its inventors.

(e) Push the Trig Menu button to activate the TRIGGER menu.
(f) Set TRIGGER Source to CH 1.
(g) Set the TRIGGER Mode to Auto.

2. **Oscilloscope Signal Deflection.**

We now want to observe how voltages applied to the input of the oscilloscope result in a deflection of the displayed signal.

(a) Push the Display button to activate the DISPLAY menu.
(b) Use the appropriate soft key to set DISPLAY Format to XY. The yellow trace should turn into a yellow dot at the center of the grid.

The connectors on the top of your breadboard are called banana connectors. Find two banana-to-BNC adapters and attach one to the CH 1 input of the scope and the other to CH 2. Now connect a wire from the black connector on the breadboard to the black connector of channel one's input connector on the oscilloscope. Connect a second wire from the red connector on the breadboard to the red post of channel one's input connector on the oscilloscope. When you connect the second wire, the dot on the oscilloscope should deflect to the right. Try connecting and disconnecting the wire relatively rapidly to send a "Morse code" signal.

Use the amount of deflection and the VOLTS/DIV setting to determine the EMF (voltage) from the battery. Is your result consistent with what you expect for two C cell batteries? Turn the Scale knob associated with CH 1 and observe what happens. Explain your observations.

Now reverse the two leads to the power supply. How does the oscilloscope trace deflect? Now move the banana-to-BNC adapter to the CH 2 input and the same connections as above. What happens? Why?

3. **Comparing Voltage Measuring Devices.**

The scope can act as a voltmeter, similarly to a hand-held digital voltmeter.

Set up the circuit you used to verify that resistors in series add like $R = R_1 + R_2$. Your circuit might have looked like that in Fig. 3.4.

Now connect the digital multimeter in parallel with the input of the oscilloscope, as shown in Fig. 3.5.

Turn the multimeter into a voltmeter by adjusting its mode switch to DCV.

Use the pair of "leads" to the oscilloscope input to measure the voltage difference across various elements of the circuit. Pay attention to polarity. Record your data in your lab notebook, and organize it as follows:

Circuit Element	Scope Reading (# of divisions of deflection)	CH1 VOLTS/DIV	Voltage determined by the scope	Voltage read by the DMM

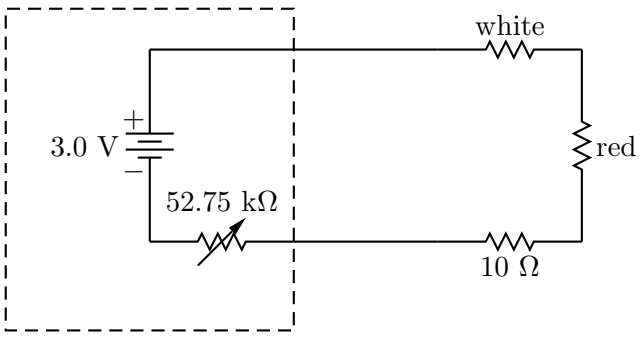

Figure 3.4. Circuit for measuring resistances in series.

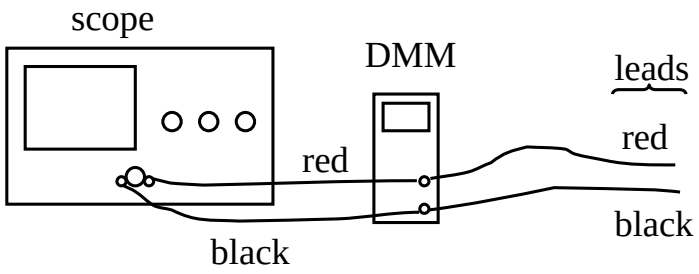

Figure 3.5. Hooking up the scope and digital multimeter in parallel.

Do the voltages measured by the multimeter and oscilloscope agree? What is the sum of the voltage differences across each of the circuit elements?

4. **Horizontal Time Sweep.**

   At this stage the oscilloscope is no more useful to us than a digital multimeter. The real power of an oscilloscope comes when we allow the oscilloscope to control the horizontal voltage for us. Change the DISPLAY Format to YT. Adjust the horizontal Scale knob to 500ms/div. The value of the scale appears at center bottom of the screen. You should now see a line slowly scanning horizontally across the screen. Use a stop watch to time how long it takes to go all the way across the screen. Is your result consistent with the 500ms/div switch setting?

5. **A Rapidly Varying Voltage.**

   We will now use a "function generator," also called a frequency generator or a signal generator, to produce a rapidly-varying signal for the oscilloscope. Use a BNC to banana adapter on the signal generator, and connect the black terminal to the ground (also black) connection on the scope and its red terminal to the CH 1 input. Other settings of the function generator are:

   VOLTS OUT — push in
   SWEEP — out position

RANGE(Hz) — push in 1k button
AMPLITUDE — fully clockwise
FUNCTION — push in sinewave button
FREQUENCY — set to 1.0
DC OFFSET — push in

Turn on the generator's power switch (the red button at the left of the front panel). Set the oscilloscope's horizontal Scale knob to 1 ms/div.

Does the scope correctly tell you that the sinusoid you are observing has a frequency of about 1 kHz? The frequency of the signal appears on the bottom right of the screen.

Note what happens to the display when you turn the Scale knob either direction. Explain your observations.

From these exercises we learn that the oscilloscope can give us a visual display of a voltage that varies rapidly in time.

6. **Trigger**

   Now let's see how the trigger controls work. Push the Trig Menu button to activate the TRIGGER menu. You should see the TRIGGER Slope set to Rising. Look at the screen at the top of the grid and you should see a white arrow point down at the trigger point of the trace. If the arrow is pointing to the left or right, then press the Set to Zero button that will bring the zero of time back to the center of the display. Is the slope of the trace at this point positive or negative? Now switch the TRIGGER Slope to Falling. What is the slope of the signal at the trigger point now? You can also adjust the Trigger Level knob that controls the arrow on the right side of the grid. The values of the trigger slope and level appear on the bottom right of the screen. How does the signal change as you adjust the trigger level? Why?

7. **Square Waves and Triangle Waves.**

   Replace the frequency generators's sinusoidal output with its square-wave output and see what you get on the oscilloscope screen. Also vary the frequency setting of the frequency generator. Then try the triangle wave output.

8. **Yet another time-varying signal.**

   Connect the output voltage of one of the small aluminum boxes to the CH 1 input. Adjust the horizontal Scale knob to 5ms/div. With the knob on the aluminum box at its maximum setting, that voltage is supposed to be a sinusoid with a frequency of 60 Hz and a "peak-to-peak" size of about 3 volts, as shown in Fig. 3.6. (That's an amplitude of 1.5 V, by the way; if you write the voltage as $V(t) = A\sin(2\pi ft)$, then $A = 1.5$ volts.)

   Are the size and frequency about right?

9. **A Mysterious Signal.**

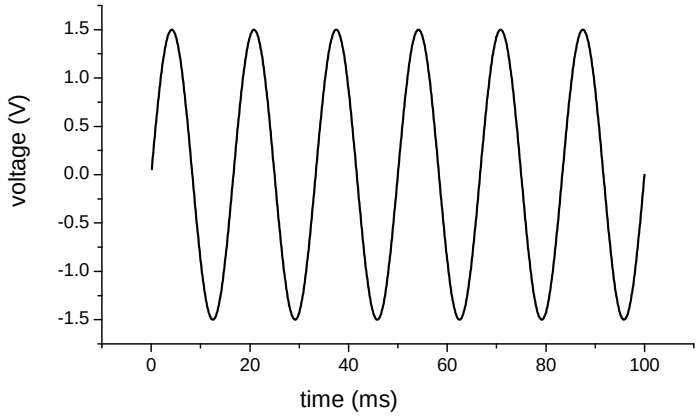

Figure 3.6. The 60 Hz sine wave.

Now disconnect the aluminum box from the oscilloscope. Connect one wire to the red post on the oscilloscope input. Hold the other end of the wire in your hand. Adjust the CH 1 Scale knob until you see a large signal on the screen. Where is this signal coming from? (Hint: what is the frequency of the signal?) Now connect another wire to the black input post and hold one wire in one hand and one wire in the other. What happens to the oscilloscope signal? Try to explain what is going on.

10. **Artistic Lissajous Figures.**

    Reconnect the 60 Hz box to the CH 1 input, and connect the sinusoidal output of the function generator to the CH 2 input. Push the Display button to activate the DISPLAY menu. Set DISPLAY Format to XY. Now the scope's time base is out of action again. The x-deflection is controlled by the voltage applied to CH 1, the y-deflection by that applied to CH 2.

    Now observe Lissajous figures produced by the oscillator and the 60 Hz x-axis voltage, for oscillator frequencies of 20, 30, 45, 60, 80, 90, 120, 150, 240, 300, 360 Hz (and higher, if you can). We suggest you start with 60 Hz. Then do 20, 30, 120, 150, 240, 300, and 360 Hz. Then return to do 45, 90 and 150 Hz, which are trickier.

    Make a sketch in your notebook of the patterns you observe for $f_x = 60$ Hz and $f_y = 30, 60, 120, 240$ Hz.

    By the way, look at the "Romer Art Machine" in the hallway, and the "operating instructions" that are posted nearby. How can you make a "fish" on the Art Machine? On the scope?

11. **More fun with the oscilloscope** (optional). If you wish to play some more with the oscilloscope, you can (for instance) measure frequencies of notes from a musical instrument or see what your own voice looks like (singing or talking), as picked up by a microphone and displayed on the scope.

3.4 Checking Out

For this informal lab no further write-up is required. Please write answers to the embedded questions in this handout in your lab notebook and a brief conclusion, and see one of the instructors for an exit interview before you leave.

Experiment 4

Capacitors

In today's lab we apply the techniques we have learned in the previous labs to a new object: the capacitor. Capacitors are to be found in almost every electronic device and, together with resistors, are the most important "passive" electronic circuit elements.

4.1 Introduction

Any set of conductors which are not electrically connected to one another can be considered a capacitor. In its simplest and most practical form a capacitor is made of just two conductors, each of which has a wire attached; the capacitor is thus a two-terminal device, just like a resistor. Unlike a resistor, however, no steady-state current can flow through a capacitor since the conductors within are not connected to one another.

For simplicity, let's consider a capacitor that is made of two flat, parallel, conducting plates, each of area A and both separated by some distance d. If we put a charge $+Q$ on one of the sheets, and $-Q$ on the other, an electric field develops in the region between the two plates, as shown in Fig. 4.1, below.

The charge resides on the sides of the plates that face one another, since this is the only configuration that ensures that the electric field within the conductors themselves is zero. Using Gauss's law, we can find the electric field between the two conductors: $E = \sigma/\epsilon_0$, where $\sigma = Q/A$ is the surface charge density on one of the plates. (We are assuming that the plate area A is quite large so that we can neglect what happens near the edge of the plates.) This is a uniform electric field.

We can also find the potential difference between the two plates by doing a line integral over the field from one plate to the other. With the plates oriented as shown in Fig. 4.1,

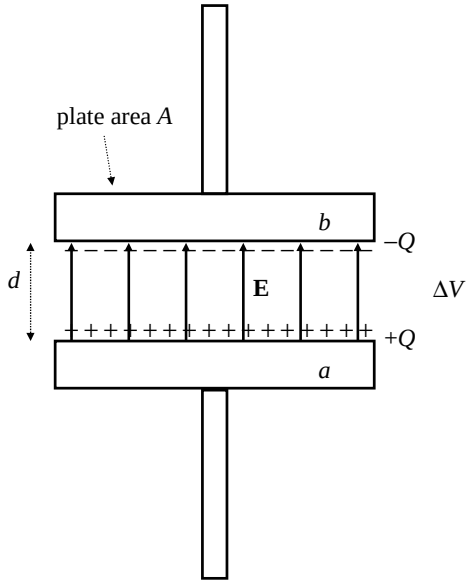

Figure 4.1. A parallel plate capacitor.

we have:

$$\Delta V = -\int_a^b \mathbf{E}\cdot\mathbf{dl} = -\int_a^b E\,dz = \frac{Qd}{\epsilon_0 A}, \tag{4.1}$$

where d is the separation between the two plates.

Observe that the potential difference ΔV is proportional to the charge we have placed on the two plates ($+Q$ on one, $-Q$ on the other): $\Delta V \propto Q$. If we call the proportionality constant $1/C$, then we have $\Delta V = Q/C$, or

$$Q = C\,\Delta V. \tag{4.2}$$

The quantity C is the *capacitance* of the two conductors. In our example we have considered what is known as a parallel-plate capacitor, and its capacitance is

$$C = \frac{\epsilon_0 A}{d}, \tag{4.3}$$

where A is the plate area and d is the plate separation. Capacitance is therefore a purely geometric factor, and can also be calculated without knowing the details of the charge on, and potential difference between, the plates. Different conductor configurations lead to different values for C.

In our water-pipe language, a capacitor behaves like a tub of water, with the area of the bottom of the tub playing the role of capacitance. The amount of charge on a capacitor is analogous to the volume of water in the tub, and the height of water in the tub is analogous

the potential difference between the two conductors. As you fill the tub with water, the height of the water goes up; and the more capacity (area) the tub has, the less the water height rises per liter of added water. Similarly, for a capacitor, the higher the capacitance, the less the potential increases per unit charge.

The units of capacitance are farads (F); 1 F is 1 C/V. The schematic symbol for a capacitor (Fig. 4.2) should bring to your mind the idea of parallel plates, even if most capacitors don't look like this any more.

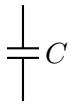

Figure 4.2. The schematic symbol for a capacitor.

Often, the space between the conductors is filled with some nonconducting material called a *dielectric*. The effect of the dielectric is to reduce the electric field within its bulk by some factor κ, which is a property of the dielectric. Different materials have different values of κ. When the electric field between the capacitor plates is reduced by a factor of κ, the potential difference between the plates is also reduced by this same factor κ (for the same charge Q on the plates). Since $C = Q/\Delta V$, reducing the potential difference increases the capacitance by κ, and thus the principal effect of placing a dielectric between the conductors of a capacitor is to *increase* the capacitance by a factor of κ. For a parallel-plate capacitor with dielectric entirely filling the region between the plates, Eq. 4.3 becomes

$$C = \frac{\kappa \epsilon_0 A}{d}. \tag{4.4}$$

4.2 Charging and Discharging

Consider the following circuit consisting of a function generator supplying a voltage $V(t)$, a resistor R and a capacitor C connected in series.

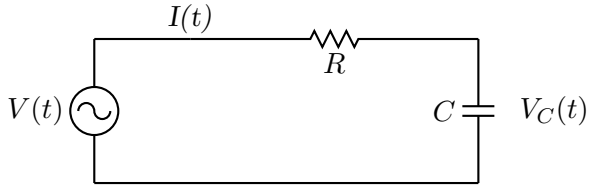

Figure 4.3. A series RC circuit attached to a time-varying voltage source.

Applying Kirchhoff's Voltage Rule to the preceding circuit gives us the following equation:

$$V(t) = I(t)R + V_C(t) = \frac{dQ}{dt}R + \frac{1}{C}Q(t), \tag{4.5}$$

where we have made use of the relationship between the potential difference across the capacitor $V_C(t)$ and the charge $Q(t)$ stored on one of the capacitor plates. Note that the current through the circuit, $I(t)$, is simply the rate of change of the charge $Q(t)$ piling up on the capacitor plate. Dividing through by R gives a differential equation for Q(t):

$$\frac{dQ}{dt} = \frac{V(t)}{R} - \frac{1}{RC}Q. \tag{4.6}$$

We will be studying two different types of behavior described by this equation.

4.2.1 Step Changes in the Voltage $V(t)$

For the first type of behavior, we will have an voltage source that jumps very quickly between two voltage values: one we call V_0; the other value will be taken to be 0.

Let's consider the following scenario. Suppose that the voltage supplied by the function generator has been 0 for a long time. (What "long" means will become apparent in a moment.) Then we know that the voltage across the capacitor will be zero; the capacitor is completely discharged. Next, let's assume that the function generator voltage suddenly jumps to the value V_0 and stays at that value. Let's call the time at which that jump occurs $t = 0$. In the differential equation for $Q(t)$, Eq. 4.6, we treat the voltage value as a constant. The solution of the differential equation with the initial condition that $Q(t = 0) = 0$ (since the capacitor is initially completely discharged) is

$$Q(t) = V_0 C \left(1 - e^{-t/RC}\right) \tag{4.7}$$

Exercise 1. Verify that Eq. 4.7 provides a solution to Eq. 4.6 when $V(t) = V_0$. You can do this by evaluating the derivative of Eq. 4.7 and substituting this result, and $V(t)$, into the differential equation.

Note that the product RC determines the time scale required for the capacitor to charge. This product is called the *time constant* for the circuit: $\tau_c \equiv RC$.

Unfortunately, we cannot measure the charge stored in the capacitor directly, but we can measure the potential difference across the capacitor (say, with the oscilloscope). The electrical potential across the capacitor is given by

$$V_C(t) = \frac{Q(t)}{C} = V_0 \left(1 - e^{-t/RC}\right). \tag{4.8}$$

Exercise 2. Using Eq. 4.8, draw a graph of $V_C(t)$ as a function of t.

Now suppose that the capacitor is completely charged. (The potential difference across it is equal to V_0.) Then suddenly the voltage drops back to 0. Let's reset our time axis so that this new drop occurs at a new $t = 0$. In this case, the charge stored in the capacitor and the potential across the capacitor begin to decrease with time:

$$Q(t) = V_0 C e^{-t/RC} \tag{4.9}$$

$$V_C(t) = V_0 e^{-t/RC} \tag{4.10}$$

Note once again that the product RC sets the time scale for the capacitor to discharge.

Exercise 3. Using Eq. 4.10, sketch a graph of the electrical potential across the capacitor as a function of time.

If you are curious, your instructors can show you how to derive these two solutions to the differential equation, Eq. 4.6.

4.2.2 Sinusoidal Voltage

For the second type of circuit behavior, we let the function generator produce a voltage that varies sinusoidally in time:

$$V(t) = V_0 \sin(\omega t) = V_0 \sin(2\pi f t) \tag{4.11}$$

where $\omega = 2\pi f$ is called the angular frequency, and f is the frequency in Hz, or cycles per second. In this case, the same differential equation, 4.6, describes the behavior of the circuit, but $V(t)$ is replaced by the sinusoidal expression given in Eq. 4.11

The general solution to the differential equation in this case is fairly complicated. If we carry out measurements on this circuit, however, we find that if we wait a time equal to a few times RC after turning on the circuit, then all the voltages in the circuit vary sinusoidally with the same frequency f, but may be shifted in phase relative to the voltage source and may have different amplitudes. We'll be looking qualitatively at how the amplitude changes as a function of frequency today.

4.3 Build Your Own Capacitor

Using the materials available in the laboratory, build your own parallel-plate capacitor. Predict what capacitance your capacitor will have, using Eq. 4.4. (If your predicted capacitance is significantly smaller than, say, 1 nF, you may have some trouble in the following

sections; you may wish to reconsider your capacitor geometry to make the capacitance bigger.) For the purposes of prediction in this lab, take the dielectric constant for wax paper to be $\kappa = 3.7 \pm 1.0$.

4.4 RC Response to a Step Change in Voltage

We wish to verify that the equations developed in Section 4.1 describe the response of an RC circuit to step changes in the applied voltage. We will then use that response to determine the capacitance of the capacitor you have built from measurements of the time constant, $\tau = RC$, of the circuit.

For a circuit with a large time constant (i.e., a minute or more) it would be possible to study the charge and discharge processes by placing an ordinary high resistance voltmeter across the capacitor and reading the potential difference every few seconds. Alternatively, the vertical input to an oscilloscope might be connected across the capacitor.

The time constants of the circuits you are provided with, however, are only a fraction of a second and the methods described above are inadequate. The technique we shall employ consists of arranging to have the capacitor go through a series of identical charges and discharges in a periodic fashion. The scope will be triggered so that similar charging and discharging cycles are superimposed on the screen.

We will use the function generator to create the switching on and off of the voltage. When set for a "square wave" output, the function generator acts like a battery with a voltage that is alternately zero and $+V_0$, switching back and forth between these two values at a rate controlled by the frequency setting on the function generator. If, for instance, the frequency is 500 Hz, then for 1 millisecond the function generator acts like the circuit on the left in Fig. 4.4, below. Then for the next millisecond, it acts like the one on the right.

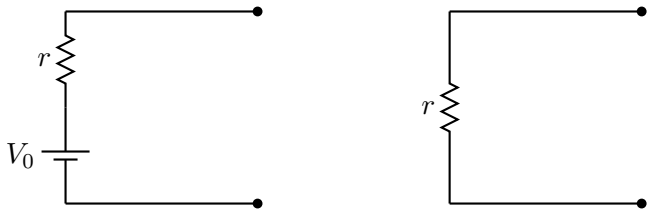

Figure 4.4. At 500 Hz, The function generator alternately acts like the circuit on the left for 1 ms, and then like the circuit on the right for 1 ms.

The size of V_0 can be adjusted with the amplitude control on the function generator. The value of the resistance r for the function generator is about 50 Ω.

Connect the circuit as shown in Fig. 4.5, below, which is essentially the same as the circuit

shown a few pages back except that the voltage source is now the function generator. **NOTE:** The ground terminal of the output of the function generator must be connected to the ground terminal of the oscilloscope.

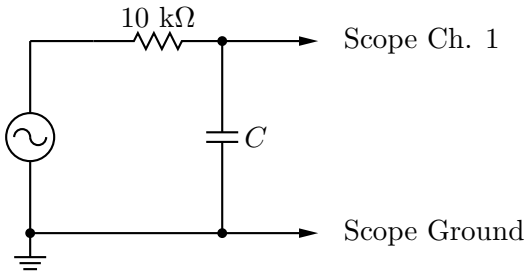

Figure 4.5. Connections for your *RC* circuit.

Set the function generator to produce a large amplitude square wave at about 3 kHz to begin with. Observe the waveform, then increase and decrease the frequency and just look at what happens. (Adjust the scope timebase and vertical sensitivity as desired.)

Set the function generator at a convenient frequency at which you can see an almost total decay of one of the exponential curves, but sufficiently spread out so that you can measure it. We recommend using the vertical Position knob on the scope to put the level that the exponential decay is approaching on the bottom line of the screen.

Measure $V_C(t)$ as a function of t for about 10 values of t. [Remember — your raw data for both V_C and t are in ticks or boxes; do not forget to note oscilloscope VOLTS/DIV and SEC/DIV settings.] You can also use the Cursor function of the scope to measure voltages at several values of t. Alternatively, you can save the scope trace to a USB "thumb drive" and then import that data into a computer.

Now change the 10 kΩ resistor shown above (Fig. 4.5) to a 100 kΩ resistor, and repeat your measurement.

4.5 Analysis

Analyze and graph at least one of the decays of the sort just described before leaving the lab. (You will show your graph and your results to one of the instructors.)

The theory developed in Section 4.1 says that the voltage across the capacitor is of the form:

$$V_C(t) = Ae^{-t/RC} \tag{4.12}$$

where A is some constant. Whence it follows that

$$\ln V_C(t) = \ln A - \frac{t}{RC} \tag{4.13}$$

So a graph of $\ln V_C$ vs. t should be a straight line. Use a spreadsheet program to plot the data with error bars and to find the slope of the "best fit" straight line. Then find C.

You have two values of C from the two sets of measurements with different values of R. If the two values are not in agreement (that is, within the combined experimental uncertainty), then something is wrong. Discuss the situation with your lab partner and/or consult with the instructors.

What is your conclusion as to what C is? Does it agree, to within an order of magnitude, with your predicted value based on the geometry of your capacitor?

4.6 Sinusoidal Response (optional)

Now use the same RC circuit with your capacitor and the 10 kΩ resistor, but with the function generator set to produce sine waves. Turn the amplitude control to its highest setting and leave it there.

We will measure the amplitude of the potential across the capacitor as a function of frequency with the following circuit (Fig. 4.6).

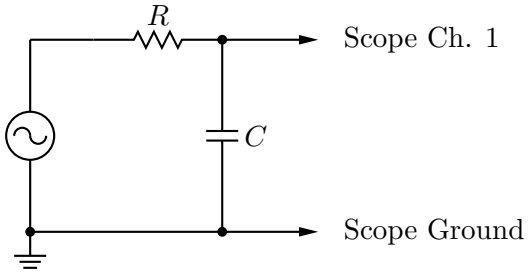

Figure 4.6. Measuring the frequency dependence of an RC circuit.

First, make a few quick observations at "high" and "low" frequencies to see the general trend of the behavior. Then judiciously select 10 frequencies (between 100 Hz and 100 kHz) for more careful measurements. Explain qualitatively why you see the behavior you do.

Plot your results for the amplitude as a function of frequency on a log-log plot. What is the amplitude (as a fraction of its maximum value) at the frequency $f = 1/2\pi RC$? From this, and your graph, do you see why this is called the "corner frequency"?

Experiment 5

Faraday's Law and Induction

CAUTION: The strong magnets we use can destroy a watch. We suggest removing watches to begin with and placing them in a safe location. Ignore this suggestion at your own risk.

5.1 Faraday's Law Introduction

It is possible to induce an EMF ε in a coil of wire by changing the magnetic flux Φ_A passing through the coil that bounds an area A. This laboratory provides a test, both qualitative and quantitative, of some of the ideas inherent in Faraday's law and Lenz's law:

$$\varepsilon = -\frac{d\Phi_A}{dt} \tag{5.1}$$

where

$$\Phi_A = \mathbf{B} \cdot \hat{\mathbf{n}} A = BA\cos\theta \tag{5.2}$$

for a uniform magnetic field $\mathbf{B}$. Here θ is the angle between $\mathbf{B}$ and the normal to the plane of the coil $\hat{\mathbf{n}}$.

Note that Φ_A is equal to BA if $\mathbf{B}$ is parallel to $\hat{\mathbf{n}}$, or $-BA$ if in the opposite direction (we have to denote, arbitrarily, one direction or the other as the direction of $\hat{\mathbf{n}}$), or equal to zero if B is perpendicular to $\hat{\mathbf{n}}$. If the coil has N turns, then the net EMF will be N times the EMF for a single loop of wire:

$$\varepsilon = -N\frac{d\Phi_A}{dt}. \tag{5.3}$$

5.2 Determining B field strength from the induced EMF

5.2.1 "One-Shot" Measurements

For *this* part, only a very rough estimate of B need be obtained; if you obtain a result you can trust to within a factor of two, you're doing OK.

For the experiment you will have available an oscilloscope, a permanent magnet which has a reasonably uniform magnetic field between its pole-faces, a coil of N turns and cross-sectional area A (both given), a motor drive for rotating the coil in the field, and a stopwatch. Use one of the coils for which data are given (see Table 5.1), and record its identification number. Also record the identification number (the "ACPL" number) of the *magnet* you use.

Coil #	N	A (cm^2)	Coil #	N	A (cm^2)
1	50	1.51	10	100	3.15
2	50	1.51	11	200	3.40
3	100	1.60	12	200	3.40
4	100	1.60	13	50	5.96
5	200	1.78	14	50	5.96
6	200	1.78	15	100	6.12
7	50	3.02	16	100	6.12
8	50	3.02	17	200	6.47
9	100	3.15	18	200	6.47

Table 5.1. Coil data.

Oscilloscope Initial Settings

1. Press the Run/Stop button until the word "Stop" appears at the top center of the screen.
2. Set the vertical Scale to 200mV/div.
3. Set the horizontal Scale to 100ms/div.
4. Press the Single button until the word "Ready" appears at the top of the screen.

Connect the coil directly to CH 1 of the oscilloscope. Set the Coupling for that channel to DC. You will want to experiment with the horizontal and vertical scale settings. Carefully insert the coil between the poles of the magnet and orient the coil so that the plane of the loop is perpendicular to the magnetic field. Quickly pull the magnet away from the coil. Observe the sign of the induced EMF and its approximate magnitude. Do this a few

times and write down your observations. Remember to press the Single button to rearm the oscilloscope each time you make a measurement. Estimate how long it took you to withdraw the magnet from the width of the peak, and measure the height of the peak for the size of the EMF. Now, using the given values of N and A, make a rough estimate for B, the magnitude of the magnetic field. (Remember that SI units must be used; your result for B will come out correctly in teslas only if voltages are in volts, areas in square meters, and times in seconds.)

Now vary the orientation of the coil in its initial position between the pole faces, and again withdraw the coil. Observe qualitatively how the size of the EMF varies with the angle of orientation, and with the speed of withdrawal of the coil. Also notice what happens when you quickly move the coil into (instead of out of) the region between the pole faces. Write down all of your observations.

5.2.2 A Rotating Coil — A Simple Electrical Generator and a Measurement of Magnetic Field Strength

If the coil is rotated with an "angular frequency" ω (radians/second), while located in a steady magnetic field, the flux will vary sinusoidally with time:

$$\Phi_A = BA \sin \omega t. \tag{5.4}$$

(For simplicity's sake, we will call $t = 0$ the time when Φ_A happens to be zero because the plane of the coil is at that time parallel to B.) Then the induced EMF is, by Eq. 5.3, N times the time derivative of this magnetic flux:

$$\varepsilon = -NBA\omega \cos \omega t. \tag{5.5}$$

That is, the induced EMF is predicted to vary sinusoidally with angular frequency ω, and with an amplitude ("center to peak") of

$$\varepsilon_{\max} = NBA\omega. \tag{5.6}$$

Oscilloscope Settings.

1. Set vertical Scale knob to 1V/div.
2. Set horizontal Scale knob to 25ms/div.
3. Press the Measure button to activate the MEASURE menu.
4. Press the top menu button and set Measure 1 Source to CH1.
5. Set Measure 1 Type to Pk-Pk.

6. Press the bottom menu button to go "Back" to the main MEASURE menu.
7. Press the second menu and set Measure 2 Source to CH1.
8. Set Measure 2 Type to Freq.
9. Press the bottom menu button to go "Back" to the main MEASURE menu.

Detailed Procedures:

1. Mount the coil on the rotator. Make sure the coil will be able to rotate freely in the magnet gap.
2. Record the frequency of rotation from MEASURE CH 1 Freq. [N.B. Don't forget that there is a difference between angular frequency (radians per second) and (ordinary) frequency (cycles per second). That is, $\omega = 2\pi f$.] Also measure the frequency of rotation using a stopwatch and the mechanical counter on the rotator. Do the two values of frequency agree?
3. Record the peak-to-peak height of the signal from MEASURE CH 1 Pk-Pk. The amplitude $\varepsilon_{\max} = (Pk - Pk)/2$.
4. Use at least 5 different rotation frequencies (ranging from very slow to very fast) and record $\varepsilon_{\max}$ (from the oscilloscope trace) as a function of rotation frequency.
5. Make a graph of $\varepsilon_{\max}$ versus angular frequency. From these data, and the N and A values for your coil (given in Table 5.1), obtain a value for B.
6. Estimate an uncertainty to be associated with this value of B. Do this by folding in results from two methods. First, consider the scatter in the data (and thereby the uncertainty in the slope). Second, take one representative data point in the middle of your data range and use the "Propagation of Errors" technique in the equation $\varepsilon_{\max} = NBA\omega$.
7. As a means of comparison, use the commercial "Hall effect" gaussmeter to measure B for your magnet, and compare it to your EMF determination of B. You may assume that the uncertainty in the gaussmeter is ± 2 in the last digit of the reading.

5.3 Minilabs on Induction

We will have only one setup of each of the following demonstrations. You may do them at any time during the lab period. In fact, it will be helpful and more efficient if you do at least one or two of them early in the lab session.

- Magnet dropped through a series of coils.

- Objects dropped through an aluminum tube.
- A magnetically-damped pendulum.
- Hand-driven generator with light bulb.

5.3.1 Falling Magnet

In this demonstration you drop a magnet through a glass tube that has six coils of wire wound around it. The coils are spaced approximately 20 cm apart.

Procedure: The apparatus here is tricky since the scope has to be programmed to make a "single-sweep" record of the trace. Please consult the Instructor before doing this minilab.

As the magnet passes each coil, it induces an EMF in that coil. (Why?)

The coils are connected to the digital oscilloscope, which "records" the signal as a function of time and then displays that signal on the oscilloscope screen. Make a careful sketch of the pattern you observe in your lab notebook.

Each of the observed "wiggles" indicates that the magnet has passed through one of the coils. Note that the size and separation of the wiggles changes with time. Explain why this happens in your lab notebook.

Repeat the experiment with the magnet reversed. Again, draw and explain what you see in your lab notebook.

5.3.2 Objects Dropped through an Aluminum Tube

There are two "identical" blue cylindrical objects in the box near the apparatus. Drop them one at a time through the long aluminum tube. Write down your observations in your lab notebook along with the answers to the following two questions:

Can you explain why the two objects behave differently?

Why does the spring scale, which supports the metal tube, read differently in the two cases?

5.3.3 Magnetically Damped Pendulum

1. Move the big magnet far away from the pendulum, then start the pendulum swinging with a small angular displacement (about 10–20°). Observe that the pendulum swings

many times before coming to a halt.

2. Stop the pendulum. Place the magnet so the pendulum can pass easily between the pole faces.

3. Now release the pendulum from roughly the same starting displacement as you did in part 5.3.3.2. Write down your observations in your lab notebook. Can you explain what happens?

4. Hold the magnet with two hands and pull it quickly away from the pendulum. Again, write down and explain what happens.

5.3.4 Hand-Driven Generator with Light Bulb

Turn the crank on the generator to get the bulb to illuminate. Repeat the experiment with the bulb disconnected. Is it now easier or harder to turn the crank? Why?

Experiment 6

RLC Circuits (Formal)

In this lab we will put together the three "linear" passive devices with which we have worked in the past few weeks: the resistor, the capacitor, and the inductor. Remarkably, whenever capacitors and inductors appear together in a circuit, Kirchhoff's voltage rule yields the equation for a simple harmonic oscillator! One important consequence is that the circuit will display the phenomenon of *resonance*.

6.1 Introduction

6.1.1 Capacitors

Recall that the voltage across a capacitor is proportional to the charge on one of the plates:

$$v(t) = \frac{1}{C} q(t) \tag{6.1}$$

We'll use lower-case letters to represent quantities that change as a function of time. If we indicate the current $i(t)$ that flows as the capacitor is discharging, we have the following situation (Fig. 6.1):

Since the current results in the charge $q(t)$ changing with time, we have, with the signs used in the figure,

$$i(t) = \frac{dq(t)}{dt} \tag{6.2}$$

$$= C\frac{dv(t)}{dt} \tag{6.3}$$

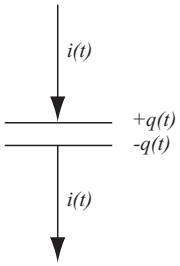

Figure 6.1.

6.1.2 Inductors

Now consider the potential difference for an inductor. The voltage across an inductor is proportional to the rate of change of current through the inductor:

$$v_L(t) = -L\frac{di(t)}{dt}. \tag{6.4}$$

Eqs. 6.3 and 6.4 suggest that capacitances and inductances are in some sense "complements" of one another:

- For a capacitor, the current is proportional to the time derivative of the voltage.
- For an inductor, the voltage is proportional to the time derivative of the current.

This situation might remind you of the relationship between velocity and position for the motion of a mass on a spring or the motion of a simple pendulum. In fact, these systems show very similar behavior (simple harmonic motion and resonance, in particular) to that of the circuit you will study today. This is due to the fact these mechanical systems are also described by Eqs. 6.3 and 6.4 (with the symbols changed, as we'll see below).

6.1.3 *LC* Oscillations — a simple case

Fig. 6.2 shows the simplest imaginable circuit containing both an inductor and a capacitor.

Suppose that at $t = 0$, the capacitor is (somehow) charged as shown. As in an "*RC*" circuit, current will start to flow, but (unlike the case of the *RC* circuit) $q(t)$ will not simply decay monotonically to zero; it will overshoot. The capacitor will become oppositely charged, then current will flow clockwise, and so on. The system continues to oscillate spontaneously, much like the (undamped) oscillations of a mass on a spring.

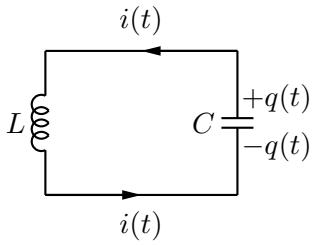

Figure 6.2.

Here is the mathematical argument: Use Kirchhoff's Voltage (Loop) Rule, and add the voltages around the loop, equating the sum to 0:

$$L\frac{di(t)}{dt} + \frac{q(t)}{C} = 0 \tag{6.5}$$

[The signs are correct, though the proof of that claim is omitted; the sign conventions for $i(t)$ and $q(t)$ are those adopted in Fig. 6.2.]

We want to focus our attention on the charge $q(t)$; so, we note that

$$i(t) = \frac{dq(t)}{dt} \tag{6.6}$$

and taking a derivative yields

$$\frac{di(t)}{dt} = \frac{d^2q(t)}{dt^2} \tag{6.7}$$

Using Eq. 6.7 in Eq. 6.5 gives us

$$L\frac{d^2q(t)}{dt^2} = -\frac{1}{C}q(t) \tag{6.8}$$

But this is an old friend, the Simple Harmonic Oscillator equation describing a mass on a spring:

$$m\frac{d^2x(t)}{dt^2} = -kx(t) \tag{6.9}$$

Comparing Eqs. 6.8 and 6.9, you might well choose to describe L as an inertia ("mass") term for a circuit. Just as a mass on a spring overshoots the equilibrium position (because of its inertia – Newton's First Law), the charge through a coil "tends to keep going" — Faraday's law inhibits sudden current changes just as Newton's First Law inhibits sudden velocity changes. Similarly, the reciprocal of the capacitance plays the role of a spring constant. Since you know the solution to Eq. 6.9 (simple harmonic oscillation with angular frequency $\omega = \sqrt{k/m}$, sometimes called, for reasons that will become clear in this lab, the resonant frequency), you can immediately predict the solution to Eq. 6.8! Those solutions are oscillatory in time with frequency $\omega = 1/\sqrt{LC}$.

6.1.4 LCR circuit with a sinusoidal EMF

What has been omitted from the preceding theory is the role of the resistance that is inevitably present. Whereas capacitors store energy (in electric fields) and inductors store energy (in the magnetic fields), resistors dissipate energy, producing heat. If a small amount of resistance is present, we will observe a "damped" sinusoidal oscillation (a sinusoid of steadily diminishing amplitude). If a large amount of resistance is present, the overshoot characteristic of oscillation may not even be seen.

Let us add some resistance to the circuit shown in Figure 6.2 and to compensate for the loss of energy we will add a sinusoidal EMF, of adjustable frequency. Now our circuit looks like that shown in Fig. 6.3.

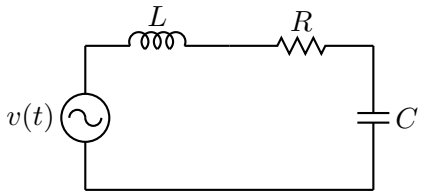

Figure 6.3.

The EMF can "drive" an oscillatory current in this circuit (alternately clockwise and counterclockwise), maintaining the amplitude of such a current at a steady value in spite of the energy being dissipated in the resistance. The frequency of this current will be the same as the frequency of the oscillatory EMF, but one might well expect (correctly) that the size of the resulting current will be greatest when the frequency of the driving EMF matches the natural resonant frequency of the LC circuit, the frequency of natural oscillations that you would predict from Eq. 6.8.

You might try the analogous experiment with a pendulum constructed from an object tied to a string. Hold one end of the string in your hand and shake it at various frequencies.

Here is another way of coming to the same conclusion. If we consider Eq. 6.3 and assume that the potential across the capacitor is given by

$$v_C(t) = V_C \sin \omega t \tag{6.10}$$

then Eq. 6.3 gives

$$i(t) = \omega C V_C \cos \omega t \tag{6.11}$$

Equation 6.11 tells us two things. First, the amplitude of the current is related to the amplitude of the oscillating EMF $I = \omega C V_C$. This looks something like Ohm's Law, with $1/(\omega C)$ playing the role of resistance. In the jargon of electronics, $1/(\omega C)$ is called the reactance or impedance of the capacitor. The crucial point is that the impedance of the capacitor varies with frequency. For high frequencies, for which the current is rapidly

oscillating in time, the capacitor does not have a chance to charge or discharge very much, and its effect (impedance) in the circuit is small. For very low frequencies, the capacitor has a very high impedance and prevents very slowly changing currents from flowing in the circuit. Secondly, there is a phase difference of 90° between the current and the potential difference for a capacitor.

Exercise 1.

- Show that $1/(\omega C)$ has units of ohms.
- Sketch a graph of $v(t)$ and $i(t)$ from Eqs. 6.10 and 6.11.

Now let's look at an inductor using Eq. 6.4. Suppose that the current varies sinusoidally with time

$$i(t) = I \sin \omega t \tag{6.12}$$

Then Eq. 6.4 tells us that

$$v_L(t) = \omega L I \cos \omega t \tag{6.13}$$

Again we see that there is a 90° phase difference between the current and the potential across the inductor. We also see that the amplitude of v_L is proportional to the amplitude of $i(t)$: $V_L = \omega L I$. The product ωL gives us the "impedance of the inductor". This impedance is high at high frequencies, because the inductor strongly opposes the rapidly changing current. For low frequencies, the inductor has almost no effect.

Look again at Fig. 6.3. A capacitor acts "like" a resistance of size $1/(\omega C)$, and an inductor "like" a resistance of size ωL, and there is a 180° phase difference between the potential across the inductor and the potential across the capacitor. At some intermediate frequency, where those two "resistances" are of equal size, then the "blocking" effects of the L and C cancel each other out, then the maximum (amplitude of) current can flow.

This frequency-dependent response is called RESONANCE. The frequency where the cancelation occurs is called the resonant frequency (denoted f_0), and the corresponding angular frequency is denoted by ω_0. From our arguments, we predict that the resonant frequency satisfies the following condition:

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{6.14}$$

CAUTION: $\omega_0 \neq f_0$ because $2\pi \neq 1$.

Exercise 2. Suppose $L = 10$ mH (10^{-2} H) and $C = 0.001$ μF. Find the numerical value of the resonant frequency f_0.

6.2 The Experiment: LCR Series Resonance

We will want to use the oscilloscope to examine the potential difference across the inductor and the capacitor as well as across the resistor. In order to carry out these measurements, we need to set up the oscilloscope in its so-called differential mode, in which the display is proportional to the potential difference between the connection to the CH1 input and the connection to the CH2 input. (We need to use this mode because we can only have one "ground" point in a series circuit.)

First, connect a wire of one color to the CH1 input (red terminal) and a wire of a different color to the CH2 input (red terminal).

Use the following oscilloscope settings:

- Press the yellow 1 button to activate CH 1 menu. Make sure Probe is set to 1× Voltage.
- Press the blue 2 button to activate CH 2 menu. Make sure Probe is set to 1× Voltage.
- Adjust both CH 1 and CH 2 vertical Scale knobs to 200mV/div. It is important that both channels be on the **same** scale.
- Adjust the horizontal Scale knob to 5μs/div. Remember $\mu s = 10^{-6} s$.
- Press the red Math button to activate the MATH menu.
- Set MATH Operation to "-".
- Set MATH Sources to CH1-CH2.
- Press the Trig Menu button to activate TRIGGER menu.
- Set TRIGGER Source to Ext.
- Set TRIGGER Mode to Auto.

Use the following function generator settings:

- SINE WAVE output
- VOLTS OUT - OUT position for 0-20 V setting
- all other push buttons in the OUT position
- AMPLITUDE - fully clockwise

Connect the the SYNC output of the function generator to the Ext Trig of the oscilloscope using a coaxial cable. This allows the the function generator to tell the scope when to trigger.

Set up the following circuit (this is really just like Fig. 6.3):

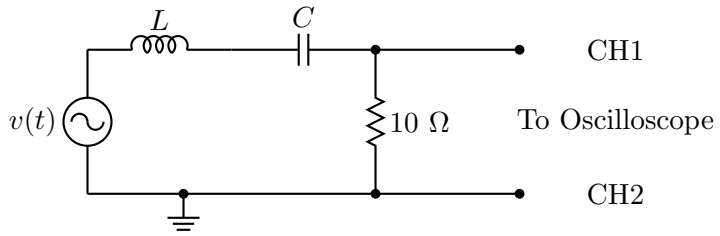

Figure 6.4.

For L, use the inductor provided. For C, use a "capacitance substitution box," set initially at 0.001 μF. Using the value of L you found earlier, make a rough estimate of the expected resonant frequency. (You should have done this in Exercise 2).

Set the Function Generator to produce a sinusoidal EMF and look for the resonant frequency. Vary the Function Generator's frequency until you find the resonance. Once you have it, admire the resonant character of the circuit's response by varying the frequency back and forth through f_0.

Measure f_0 with the oscilloscope. Now repeat the experiment for about eight other C values in the range 0.001 μF to 0.22 μF, finding f_0 for each capacitance. Be sure to record the uncertainties in your frequency measurements, which will not necessarily be the accuracy with which the counter counts! Ask an instructor if you're uncertain about your uncertainty.

Measure the C values in the substitution box using a digital capacitance meter. [It is more accurate than the nominal values printed on the substitution box; the principles on which this capacitance meter operates are the similar to those used in Lab 4, "Capacitors."]

From Eq. 6.14, it follows that a graph of C vs. $1/f^2$ should be a straight line, from whose slope you can find L. Make such a graph, and find L.

6.3 Two Puzzles

6.3.1 Initial Measurements

Pick one of the eight capacitance values you used in Sec. 6.2 that has a capacitance $<$ 0.015 μF. Replace the 10Ω resistor with a 1 kΩ one. With the CH1 and CH2 wires connected to opposite ends of the 1 kΩ resistor, set the function generator frequency so that it is at the resonant frequency for this capacitor. **Be sure to use the 1 kΩ resistor for this part!** Then move the CH1 and CH2 wires to determine successively the amplitude of:

- the emf from the function generator;
- the potential drop across the capacitor;
- the potential drop across the inductor;
- and the potential drop across the resistor.

There should be two puzzling aspects of your results:

1. Kirchhoff's Voltage Rule appears to be violated: the emf amplitude (from the function generator) is not equal to the sum of the amplitudes of the potential drops around the circuit.
2. The amplitude of the potential difference across the capacitor (or the inductor) is larger than the amplitude of the emf from the function generator: we are getting out "more voltage" than we are putting in.

6.3.2 Resolution of the Puzzles

To resolve the first puzzle, adjust the oscilloscope to display two or three cycles of the potential across the inductor. Adjust the VOLTS/DIV (on both CH1 and CH2) so that the display nearly fills the screen vertically.

Draw a careful graph of the signal you observe.

Now without changing any of the oscilloscope or function generator settings, move your CH1 and CH2 wires to observe the other three signals listed in Sec. 6.3.1. (In carrying out these measurements, it is important to keep the order of the CH1 and CH2 wires the same as you move around the circuit. For example, if you use the CH1 wire connected to point D in Fig. 6.5, below, and the CH2 wire connected to point E to measure the potential difference across the resistor, then when you measure the potential difference across the

capacitor, CH1 should be connected to point B and CH2 to point D.) Record these other three signals on your graph.

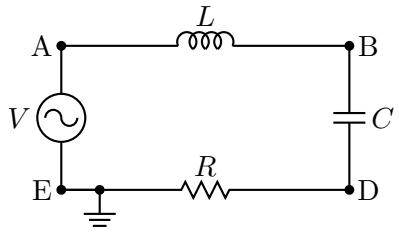

Figure 6.5.

Does Kirchhoff's Voltage Law describe the behavior of the circuit at each instant of time?

The lesson here is that your amplitude measurements did not take into account the phase differences in the circuit: the various potential differences do not reach their maximum values at the same time.

Using the function generator emf as your time reference, what are the phase differences of the other potential differences in the circuit? Sketch the V vs. t you observe on the oscilloscope for each of the devices in the circuit including the function generator. Note the difference in times when each device reaches a peak.

The resolution of the second puzzle, the large amplitude of the potential difference across the capacitor (or the inductor) relative to the emf value is a little harder to explain. Can you speculate what might be going on using an analogy with resonance in a pendulum?

Experiment 7

Properties of Light

7.1 Snell's Law

In this part of the lab, you will investigate the fundamental relationship between incident and refracted light beams. This is the foundation for building more complicated refractive optics such as lenses and optical instruments consisting of combinations of lenses. Snell's Law states that for light incident at an angle θ_1 on a smooth interface between two materials of indices of refraction n_1 and n_2, the angle θ_2 of the refracted light is related to θ_1 by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{7.1}$$

where the angles are measured between the light rays and the normal to the interface (see Fig. 7.1).

7.1.1 Procedure

1. Place the D-shaped semicircle of gelatin ("stiff water") within its plastic holder on top of the ray table (Fig. 7.2). Place the gelatin in such a way that the straight side is aligned with the COMPONENT line on the paper, and so that the midpoint of the straight side sits at the center of the ray table, as shown in Fig. 7.1. Set the laser so that the light beam shines through the circular "wall" of the gelatin and exits at the midpoint of the straight side (see Fig. 7.1, below). The light will enter the gelatin normal to its surface. You will need to set the height of the laser (by propping it up on a book or some such object) so that you can see the entrance and exit of the laser light.

2. Rotate the ray table until you discover the angle θ_C for total internal reflection for water. (Make sure that the beam is exiting at the midpoint of the straight side.)

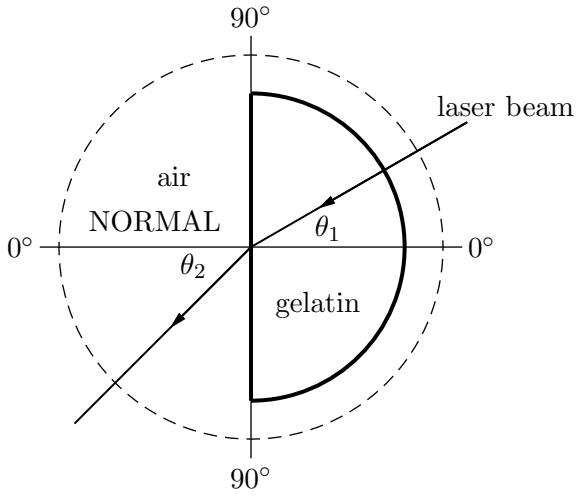

Figure 7.1.

Figure 7.2. Basic Optics Ray Table

What is it? Use it to determine the index of refraction of gelatin from the relation $\sin\theta_C = 1/n_G$.

3. Record the incident and refracted beam angles, θ_1 and θ_2, respectively, starting with $\theta_1 = 0$ and increasing θ_1 in 5° increments for as many data points as you can. Make sure to read the θ_2 values relative to the normal, i.e., relative to the 180° line. Plot your data to see if Snell's Law describes your results. If so, use it to get a value for n_G. How does this value compare with the known value $n_W = 1.333$, the index of refraction for water?

7.2 Inverse Square Law Behavior

Here you will explore the phenomenon that light from a point source propagates outward uniformly in all directions about the source. In the absence of any lenses, mirrors, interfaces, etc., light will propagate outward from a point source in a straight line, filling a spherically symmetric volume. If the total energy per unit time produced by the source is constant, the intensity (energy per unit area per unit time) must fall off with distance from the source. In fact, since the area of a spherical shell surrounding the source increases as the square of the distance r from the source, the intensity I is expected to exhibit "inverse square law behavior" when measured as a function of r.

Exercise

Write down a mathematical expression that relates I to r as described in words in the previous sentence. Draw a picture if this helps.

Experiment and Analysis

On an optical bench, align a light bulb 10 cm in front of a photo sensor (light detector), as shown in Fig. 7.3. The detector is designed so that it will give an output voltage proportional to the intensity of the light hitting the sensor so long as the voltage is less than 300 mV. That is, for low intensities, the detector responds linearly to the intensity. We will want to conduct this experiment in the linear region of the detector. To make sure this happens, place a holder that contains a collection of light absorbers (these are actually just partially darkened overhead transparencies cut into 2 inch squares) between the bulb and the detector so that the output reading is close to but not more than 300 mV. The absorber (filter) allows only a fraction of the light to penetrate through it. It should be positioned very close to the bulb.

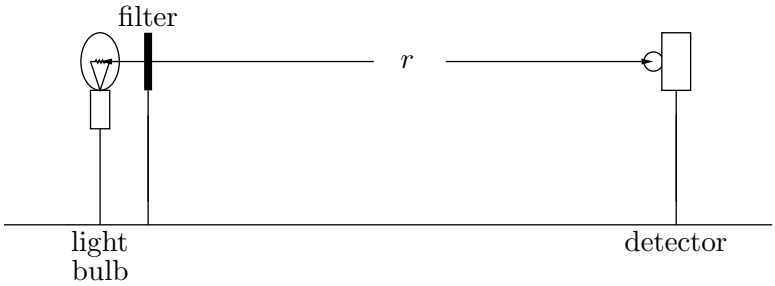

Figure 7.3. Note: distance r is measured from light source (filament) to sensor.

Examine qualitatively what happens to the intensity when the source-detector spacing r is gradually increased. Take quantitative measurements of the detector output voltage V vs.

r for at least 10 readings between 10 cm and 80 cm. Take more data points closer to the bulb since that's where the change in V is the greatest. You will have to subtract off any nonzero background voltage (i.e., the voltage when your bulb is off) from the V readings, so make sure to record the background in your notebook, too. First, make a plot of V (corrected) vs. r. Then employ the commonly-used method of straight-line graphing to plot your data in such a way that you observe a straight line. A log-log plot will help with this. (Why?) How linear are your results? What is the slope, and how does it compare to the expected value of slope from the expression you determined in the Exercise? Can you identify reasons for any discrepancies by looking carefully at your plot?

For your exit interview: We will ask you to show all calculations and graphs.Please make sure to answer all questions posed above.

Experiment 8

Geometric Optics (Formal)

If a point source is a distance s from a "thin" lens, then light diverging from the source will be refracted by the lens and will converge at a distance s' from the lens (see Fig. 8.1). Here s and s' are related to the focal length, f, of the lens by the Gaussian lens equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \tag{8.1}$$

where f is a length characteristic of the particular lens used. Its value depends upon the radii of curvature of the spherical surfaces of the lens and on the material of which the lens is made; i.e., its index of refraction. (Note: in the drawing we assume $s > f$.)

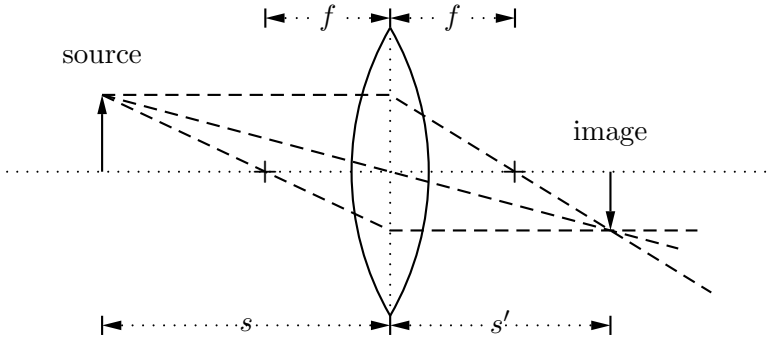

Figure 8.1.

If a screen were placed at s', then a bright spot would appear on the screen. If, instead of a point source, we had an extended source a distance s from the lens, then a focused image of this extended source would appear on the screen at a position s'. Indeed, if we placed a photographic plate instead of a screen at this position, we would have the makings of a (film) camera.

In today's lab, you will test the validity of equation 8.1.

Part I. If an object is very far from the lens ($s \gg f$), then $1/s \ll 1/f$.

In this case we expect from the equation that the distant object will be focused at a distance $s' = f$. To get a crude idea of the focal length of the lens you are using, find some very distant bright object (e.g., a mountain or a tree) and measure the distance from the lens to a point where the light is focused. Be sure to include with your measurement an estimate of the uncertainty associated with f.

Part II. You have been provided with a light source, a screen, an optical bench, and a meter stick. Using these, set up an experiment to test the validity of Eq. 8.1. Plot your data in such a way that, if the theory is correct, you ought to obtain a straight line graph from which you can obtain the focal length of the lens. Again, be sure to estimate the uncertainty in your measurement. Does your value deduced here agree with that obtained in Part I? While making these measurements, observe the characteristics of the image produced. Is the image larger or smaller than the original object? Is the image inverted?

Part III. Repeat Part II with a different lens with a different curvature, and hence focal length. Which lens has the larger f? For which lens are the surfaces more curved (i.e., depart further from being planes)? Which lens is "stronger"? The strength or "power" of a lens is defined by the reciprocal of its focal length. It is measured in units of diopters (= 1/meters). (If you wear glasses or contact lenses, your lens prescription is specified in units of diopters.) What are the strengths of your lenses in diopters?

Part IV. Take one of the "negative" lenses and "sandwich" it with one of the converging lenses from either Part II or Part III. Use the converging lens that is the stronger of the two; i.e., the higher power (in diopters). Use tape around the edge to hold the two lenses together and place the "composite" lens into the stand on the optics bench. Now perform a set of measurements of s and s' to determine the focal length of the composite lens. Once you have found the combined focal length, solve for the focal length of the negative lens using:

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{comb}}, \tag{8.2}$$

where one of the focal lengths f on the left hand side of Eq. 8.2 is the focal length of the converging lens you used.

Experiment 9

Interference, Diffraction and Polarization

In our last laboratory on geometric optics we treated light as if it were composed of rays. While this approximation is usually adequate for objects which are large compared to the wavelength of light, it is not adequate for describing the interaction of light with small objects. In this case, it is necessary to consider light as an electromagnetic wave. The electric and magnetic fields associated with a light wave are always perpendicular to the light propagation direction. The magnitude of the electric field associated with a beam of light propagating in the x-direction may be written as

$$E = E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right]. \tag{9.1}$$

This describes a "travelling" wave with an amplitude E_0 and a velocity equal to $f\lambda$, where f is the frequency of the light and λ is the light's wavelength.

The intensity of an electromagnetic wave is proportional to the total electric field squared (E^2). Because light is a wave, it may exhibit the property of interference. Consider what would happen if two electromagnetic waves were travelling in the same direction but with their phases shifted by 180°. That is, we let one wave have its maximum just as the other is at its minimum. The total electric field is just the sum of the electric fields associated with each beam, in accordance with the principle of superposition.

So

$$\begin{aligned} E &= E_1 + E_2 && (9.2)\\ &= E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] - E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] && (9.3)\\ &= 0. && (9.4) \end{aligned}$$

The field associated with the second beam will exactly cancel the first and there is no net field. Since there is no field, there is no intensity, and hence we would see no light. This

phenomenon is called total destructive interference. Alternatively, we could imagine two electromagnetic waves having the same amplitude and phase. Then the total field would simply double:

$$E = E_1 + E_2 = 2E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right]. \tag{9.5}$$

Because the intensity of light is proportional to the square of the electric field the intensity would quadruple. This phenomenon is called "constructive" interference.

9.1 Young's Double-Slit Experiment

Today we would like to observe these "wavelike" properties of light. To do this we will do Young's double-slit diffraction experiment. In this experiment an electromagnetic wave is incident upon two narrow apertures. We let the separation between the two apertures be d. The electromagnetic wave will exit the two slits with approximately the same amplitude and phase. If we place a screen some distance L away from our two slits, the electromagnetic waves propagating from the two slits will combine to illuminate the screen (see Fig. 9.1).

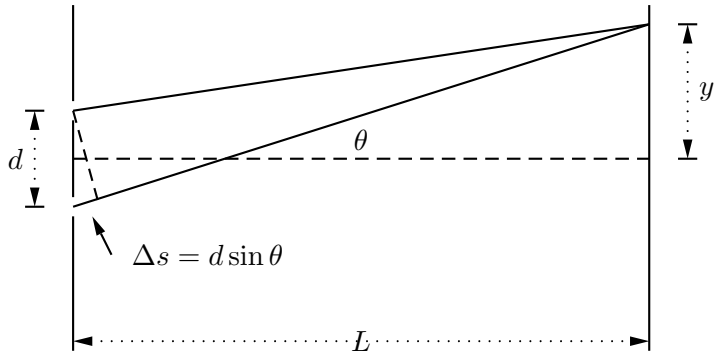

Figure 9.1.

The distance traveled by the wave from slit 2 to the screen is longer than that traveled by the wave from slit 1. The difference in the path lengths is approximately equal to

$$\Delta s = d \sin\theta \tag{9.6}$$

where θ is the angle shown in the figure.

Now, if $y \ll L$, then θ is small and $\sin\theta = y/L$. The difference in travel distances is then $\Delta s = yd/L$.

If this path difference is equal to $n\lambda$, where n is an integer, then there will be a constructive interference and we will see a bright spot on the screen. Thus the positions of the bright

spots will be at positions y_n given by

$$n\lambda = \frac{y_n d}{L} \qquad \text{or} \qquad y_n = \frac{nL\lambda}{d}. \tag{9.7}$$

The separation between two successive bright spots will then be given by

$$\Delta y = y_{n+1} - y_n = \frac{L\lambda}{d}. \tag{9.8}$$

Similarly, for points where the path difference Δs creates a phase shift of 180° there will be destructive interference and no light will be observed. These points of destructive interference will be halfway between the bright maxima. They will also be separated by a distance

$$\Delta y = \frac{L\lambda}{d}. \tag{9.9}$$

If we measure Δy, d, and L, we can in principle measure the wavelength of the incident light.

In order to perform this experiment a helium-neon (He-Ne) laser will be used. Lasers create light that propagates only in a particular direction and is monochromatic (i.e., it has only one frequency or wavelength). You will determine the wavelength of this laser light from your double-slit interference experiment.

Part I. You have been provided with a photographic plate that has a column of double slits. For our lab today we will use at least two of these double slits. For ease of measurement we recommend the second- and third-widest double slit pairs on your film; note that in the "series" of double-slits, the narrowest is actually only a single slit.

For at least these two slit pairs measure the distance d between the slits as follows: put the whole photographic plate on an overhead projector to magnify the image. Measure the slit width of the image as well as the width of the whole photographic plate and its image at the horizontal line through the slits whose width is being measured. Because the slits have a finite width, you may want to measure edge to edge and average your results for left edges and right edges to get a value for d. Repeat this measurement enough times so that you have a good idea of the uncertainty in d.

Part II. Shine your laser beam onto one of the double slits that you have measured. Place a screen at large distance L (at least a couple of meters) away from the double slit. You should observe several points of constructive and destructive interference. To get maximum sensitivity here it is important that the ambient light be dim. Measure the distance between successive minima. To get the best precision in this measurement, measure the interval over several successive minima and divide by the number of intervals. From your measurement, determine the wavelength of the He-Ne laser. Be sure to include an estimate

of the uncertainty in your measurement. (CAUTION: You will see successive brightening and dimming of the maxima as you move away from the center. This is an interference effect associated with the finite slit widths. Do not confuse it with the double-slit interference pattern.)

Part III. Repeat Part II for your second slit pair. Obtain a second value for the He-Ne wavelength. In which measurement do you have more confidence? Why?

Part IV. A remarkable prediction of our treatment of light as a wave is that the intensity at the points of destructive interference will be zero when both slits are open, but non-zero when light is arriving from either slit alone. That is — when we open the second slit, we *decrease* the light intensity at these points! See if you can observe this effect by covering and uncovering one of your slits.

Part V. Now that you know the laser's wavelength, you can reverse the process and use the observed interference to determine the spacing between the "slits". Shine your laser beam on a compact disk. Observe the bright spots due to constructive interference. (You will probably see only a few.) From the geometry of the interference pattern determine the spacing between the "grooves" on the CD.

N.B. In this case, the angles are *not* small so you need to use $\Delta s = d \sin\theta_n = n\lambda$ for constructive interference.

9.2 Malus's Law for Polarization

Certain materials (such as the polymers in our polarizers) have the ability to respond to an incident electric field by absorbing all of the light that has an electric field aligned with the polymers, and re-radiating the light that does not. We can think of light incident on a polarizer as having two components of electric field, one aligned with the polymers, and one perpendicular to that direction. (The third dimension, perpendicular to the plane of the polarizer, is unaffected and need not be considered here.) The light that makes it through the polarizer is the perpendicular component. We say that the transmitted light is *polarized* because it comes out having an E-field in a single, well-defined direction (that we might mark on the polarizer with an arrow), regardless of what its E-field orientation was to begin with. This is good news for those of us who wear polarized sunglasses since the polarizer acts somewhat as an E-field filter and cuts the transmitted light intensity way down.

To get polarized light in the first place, we can pass unpolarized light (light with many random orientations of the E-field vector, such as the light from our bulb) through one

polarizer. It turns out that in that case, half of the incident light intensity makes it through. If you shine that polarized light onto a second polarizer, the polarization of the output light is determined by the second polarizer, as discussed above. But the intensity of that light depends on the relative polarization angles of the polarizers. Malus's Law describes the overall output intensity as a function of the angle θ between the alignment axes of the two polarizers. In fact, Malus's law says that the output intensity (out of the second polarizer) and the input intensity (out of the first polarizer) are related by the factor $\cos^2\theta$.

Exercise I

Write down the mathematical expression that is described in the previous sentence.

Exercise II

Get a qualitative feel for the effect of crossed polarizers by looking through a stack of two polarizers and rotating one relative to the other. You'll probably need to do this in a well-lit room. What do you notice about the light intensity? Do your observations make sense in light of the above discussion?

Appendix A

Keeping a Lab Notebook

Keeping a good lab notebook seems like a simple and obvious task, but it requires more care and thought than most people realize. It is a skill that requires consistent effort and discipline and is worth the effort to develop. Your lab notebook is your written record of everything you did in the lab. Hence it includes not only your tables of data, but notes on your procedure, and your data analysis as well. With practice, you will become adept at sharing your time fairly between conducting the experiment and recording relevant information in your notebook as you go along.

You want all this information in one place for three main reasons, and these reasons continue to be valid even after you leave the introductory physics laboratory. (That is, even — or rather, especially — practicing scientists keep lab notebooks.) First, your lab notebook contains the information you will need to write a convincing report on your work, whether that report is for a grade in a course or a journal article. Second, you may need to return to your work months or even years after you have finished an experiment. It is surprising how often some early experiment or calculation is important in your later work. Hence you need a reasonably complete account of what you have done. Third, your lab notebook is also the source to which you turn in case someone questions the validity of your results. (You may have heard about the famous David Baltimore case of alleged scientific fraud, in which the lab notebook of one of Baltimore's collaborators was the subject of careful scrutiny.)

Your notebook therefore serves two purposes that may not be completely compatible with each other. On one hand, you should write things down pretty much as they occur and before you have a chance to forget them, so that you have a complete record of your work in the lab. On the other hand, your notebook should be reasonably neat and well-organized, partially so you can find things and partially so that if anyone questions your results, not only will they be able to find things, but the layout of your notebook will suggest that you investigated the problem carefully and systematically.

You should use a bound lab notebook (that is, not a loose leaf notebook). So-called quadrille

notebooks (with rectangular grids on each page) are particularly handy for making graphs and tables. We strongly recommend that you leave every other sheet in your lab notebook free, so that you can jot down additional comments and/or add graphs onto those blank sheets after the fact. If you wish to add a graph done on a computer or a graph done on regular graph paper to the notebook, you may simply tape or glue the graph into your notebook.

Next, we will discuss some of the information that goes into your lab notebook.

Introduction

You should begin each new experiment on a fresh page in your notebook. Leave some room for pre-lab lecture notes. Start with the date and brief title for the experiment — just enough to remind you what that section of your notebook is about. Then give a list of the equipment, identifying large pieces of equipment with manufacturer's name and the model. For large pieces of equipment, record the serial number, too. With this information, you can repeat the experiment with the identical equipment if for some reason you are interrupted and have to the return to the experiment much later. Or, if you are suspicious of some piece of equipment, having this information will let you avoid that particular item.

Sketch of the Setup

Also make a quick sketch of the setup, or a schematic diagram (for electronics). Schematics will be especially helpful when you will be connecting various pieces of electronic equipment together in mildly complicated ways. Also in optics experiments, ray diagrams are useful to keep track of the paths of various light rays.

Outline of Methods

Next, give a short paragraph noting the main goal of the experiment and outlining how you expect to carry out your measurements. This should not be too detailed, since you will probably modify your procedure as you go along. But this opening paragraph will help you settle in your own mind what you do to get started. Particularly as the semester goes on and you develop more and more of the experimental procedure yourself, you will find yourself modifying your initial procedure, discovering additional variables that should be recorded and revising your approach. So you don't want to get too locked in to one format. But you also should avoid writing down data or procedures in the nearest blank space, or you'll be cursing yourself when you look for those pieces of information later. So the cardinal rule of keeping a lab notebook is this: give yourself plenty of space. Doing so makes extending tables or descriptions of procedure easy, and typically also makes your notebook easier to

read. If you find that you haven't allotted enough space for a table, feel free to start it over on the next page — labeling the new table, of course, and making a note at the old table directing you (or someone else) to the new one. Using only the odd-numbered (or even-numbered, if you're left-handed) pages also works well; the blank facing page can be used later to reduce data, where you can see both the raw data and the reduced data, or for graphs. Or you can use all the pages, but start out using only the top half of each page.

Procedure

In recording your procedure, write in complete sentences and complete paragraphs. This is part of the discipline required for keeping a good lab notebook. Single words or phrases rapidly become mysterious, and only with a sentence or two about what you're measuring, such as the period of the pendulum as a function of length, will you be able to understand later on what you did. Give more details where necessary, if for example the lab manual does not give a more detailed procedure or if you depart from the procedure in the manual.

Numerical Data

When recording numerical data, keep your results in an orderly table. You should label the columns, and indicate the units in which quantities are measured.You should also indicate the uncertainty to be associated with each measurement. If the uncertainty is the same for a certain set of data, you can simply indicate that uncertainty at the top of the column of that data.

You will need at least two columns: one for the independent variable and one for each dependent variable. It's also good to have an additional column, usually at the right-hand edge of the page, labeled "Remarks." That way, if you make a measurement and decide that you didn't quite carry out your procedure correctly, you can make a note to that effect in the "Remarks" column. (For example, suppose that you realize in looking at your pendulum data that one of your measurements must have timed only nine swings instead of ten. If you indicate that with, say, "9 swings?" you could justify to a suspicious reader your decision to omit that point from your analysis.)

Do not Erase

It is always a good idea to record data, comments, and calculations in ink rather than in pencil. That way, you avoid the temptation to erase data that you think are incorrect. You never should erase calculations, data, comments, etc., because the original data, calculation, and so on may turn out to be correct after all, and in any case you want to keep a complete record of your work, even the false starts. If you believe that a calculation, for example is

wrong, it is better to draw a line through it and make a note in the margin than to erase the calculation. You can always make things neat in your report.

Sequences of Measurements

You will often be performing experiments in which you have two independent variables. Usually in such experiments you fix the value of one independent variable and make a series of measurements working through several values of the other variable. Then you change the value of the first variable and run through the measurements with the other variable again; then you change the first independent variable again, make another set of measurements, and so on. It's usually easier to set up this sort of sequence in your notebook as a series of two-column tables (or three columns with 'Remarks") rather than a big rectangular grid. "Title" each table with the value of the independent variable that you're holding fixed, and keep the format of all of the tables the same.

Comment on Results

Once you have completed the experiment and performed any necessary calculations in the notebook, you should look back to the main goal and write down to what extent it was achieved. If, for example, you were making a measurement of g, you should include a clear statement of the value of g along with its uncertainty. Be aware that there are often secondary goals as well (to become familiar with a particular physical system or measurement technique, for example). Comment on your success in attaining these goals as well. This serves as a statement of conclusion and gives you the chance to make sure the lab was completed thoroughly and to your satisfaction.

Guidelines to keeping a good notebook

1. Bound Notebook: No spiral bound, loose leaf or perforated page notebook. Lab notebooks are a *permanent record* of the work done in lab. The integrity of the notebook should not be comprised by tearing out pages.
2. Keep a record. Write down names, title, time, places and dates.
3. Be generous with use of pages. Start each experiment on a fresh page. leave some blank pages between experiments in case you need to add tables or graphs.
4. Do not erase, redact or scribble out possible mistakes. Draw one single line through any value or calculation you suspect may be wrong.
5. Define terminology and variables with units.

6. Sketch the experimental setup. Label the relevant parts and indicated measured quantities. List equipment used.

7. Be complete. Every experiment has a introduction (pre-lab note), procedure, analysis, results and summary. Make sure all tables, graphs, diagrams and calculations are in your notebook for reference.

8. Annotate! Each section of experiment should begin with a paragraph discussing what the section is about. Do not leave it to a reader to guess.

9. Be organized, neat and legible. The reader should not to have struggle to decipher your notes. Don't be cryptic.

10. Answer all questions and exercises. Use your answers to questions build the discussion of your formal report or exit interview. Always answer questions and exercise from a physics point of view.

Appendix B

Graphical Presentation of Data

B.1 Introduction

"Draw a picture!" is an important general principle in explaining things. It's important because most people think visually, processing visual information much more quickly than information in other forms. Graphing your data shows relationships much more clearly and quickly, both to you and your reader, than presenting the same information in a table.

Typically you use two levels of graphing in the lab. A graph that appears in your final report is a "higher-level" graph. Such a graph is done neatly (and almost always with a graphing program), following all the presentation guidelines listed below. It's made primarily for the benefit of the person reading your report. "Lower-level" graphs are rough graphs that you make for your own benefit in the lab room; they're the ones the lab assistants will hound you to construct. These lower-level graphs tell you when you need to take more data or check a data point. They're most useful when you make them in time to act on them, which means that you should get in the habit of graphing your data in the lab while you still have access to the equipment. (That's one reason, in fact, that we recommend that you leave every other sheet in your lab notebook free, so you can use that blank sheet to graph your data.) In graphing your data in the alb, you don't need to be too fussy about taking up the whole page or making the divisions nice. You should label the axes and title the graph, though.

Flaky data points show up almost immediately in a graph, which is one reason to graph your data in the lab. Skipping this low-level graphing step can allow problems in the data collection to propagate undetected and require you to perform the experiment again from the beginning. Graphing each point as you take it is probably not the best idea, though. Doing so can be inefficient and may prejudice you about the value of the next data point. So your best bet is to take five or six data points and graph them all at once.

Graphing your data right away also flags regions in your data range where you should take more data. Typically people take approximately evenly-spaced data points over the entire range of the independent variable, which is certainly a good way to start. A graph of that "survey" data will tell you if there are regions where you should look more closely; regions where you graph is changing rapidly, going through a minimum or maximum, or changing curvature, for example. The graph helps you identify interesting sections where you should get more data, and saves you from taking lots of data in regions where nothing much is happening.

B.2 Analyzing your Graph

"Graphical data analysis" is usually a euphemism for "find the slope and intercept of a line." You will find this semester that you spend a lot of time redrawing curves by employing the "method of straight line graphing" so that they turn into straight lines, for which you can calculate a slope and an intercept. This process is so important that, although we have a fond hope that you learned how to do this in high school, we're going to review it anyway.

Presumably you have in front of you some graphed data that look pretty linear. Start by drawing in by eye the line that you think best represents the trend in your data. An analytical procedure exists to draw such a line, but in fact your eyeballed line will be pretty close to this analytically-determined "best" line. Your job now is to find the slope and intercept of that "best" line you've drawn.

Next we tackle the question of finding the slope and intercept of that line. As usual, we will assume that the line is described by the equation

$$y = mx + b \tag{B.1}$$

where m is the slope of the line and b is the y-intercept.

Two points determine a line, and a line is also described completely by its slope and intercept. (This should make a certain amount of sense. You put in two pieces of information, you get out two pieces of information.) Your first task is therefore to choose two points on your line. These two points describe the line, so they need not (and most likely *will* not) be data points. They should be far apart on the graph, to minimize the effects of the inevitable experimental uncertainty in reading their locations from the graph paper. The two points should also be located at easy-to-read crossings on the graph paper. Mark each of those points with a heavy (but not too large) dot and draw a circle around the dot. Read the coordinates of each point off the graph.

The slope of the line is defined as the change in y (the vertical coordinate) divided by the corresponding change in x (the horizontal coordinate). (You may know this in some other

form, such as "rise over run.") To calculate the slope, use

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}, \tag{B.2}$$

substituting your values for the points (x_1, y_1) and (x_2, y_2). For example, if your points are $(1.0 \text{ sec}, 8.8 \text{ m/sec})$ and $(6.0 \text{ sec}, 46.3 \text{ m/sec})$, then $m = 7.5 \text{ m/sec}^2$. (Notice that the units of m are the units of "rise-over-run.") Now that you have the slope, find the intercept from

$$\text{intercept} = b = y_1 - mx_1. \tag{B.3}$$

That is, you can read b directly off off the graph, or you can use the slope and one point to determine b. Use either point for (x_1, y_1). Both lie on the line, so either will work. In the example above, we get that $b = 1.3$ m/sec. (Notice that the units of b are those of the y variable.)

Once you have determined the values of m and b from the graph, you can quote the equation for your straight line. For example, if $m = 7.5 \text{ m/sec}^2$ and $b = 1.3$ m/sec, then the equation of your straight line is

$$y = (7.5 \text{ m/sec}^2)x + 1.3 \text{ m/sec}. \tag{B.4}$$

This equation gives a **complete** description of the line and the job is done.

B.3 Uncertainty Bars

Individual data points plotted on any graph should include uncertainty bars (sometimes misleadingly called "error bars") showing the uncertainty range associated with each data point. You should show both vertical and horizontal uncertainty bars, if the uncertainties are large enough to be visible on the graph. (If they aren't large enough, you should mention this in your report so we don't think you've forgotten them.) You can draw uncertainty bars by indicating the "best guess" value (typically the measured value or average of several measurements) with a dot, and drawing an "I-bar" through the dot with its length indicating the range in the uncertainty. When you use Excel or the Graphical Analysis data analysis package, this step can be done for you — with severe limitations. Such a package will typically only determine error bars by considering the scatter of the individual data points about the best-fit straight line. While this is helpful in providing a consistency check for the data, it does not tell the complete story of the uncertainties in your data. That is, unless you use a more advanced feature of such an analysis program, it has no way of knowing about the uncertainties that were inherent in your measured values because of the measurement apparatus. Only you can decide how accurately you used the meter stick, or how quickly you were able to react when starting and stopping a stopwatch. You will not always be expected to put error bars on all of your plotted points, but you should know how it is done and be able to apply it to the first lab.

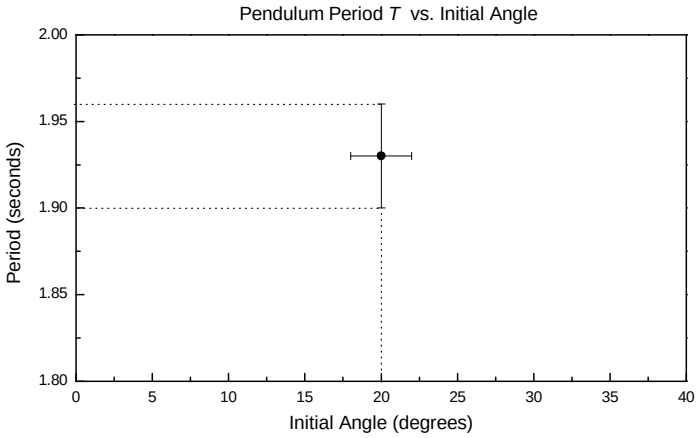

Figure B.1. A graph of one data point of the pendulum period as a function of angle, showing uncertainty bars for both variables.

An example of such an uncertainty bar is shown in Fig. B.1. The single data point plotted corresponds to a measured pendulum period T of 1.93 sec $\pm$ 0.03 s for an initial release angle θ of $20° \pm 2°$. The dotted lines are not part of the graph, but are included to show you how the point and the uncertainty bars are related to the axes. (Notice also that the T-axis does not begin at $T = 0$.

B.4 Graphical Presentation Guidelines

Use these guidelines for "higher-level" graphs.

1. Draw your hand-plotted graphs in pencil; mistakes are easy to make. If you wish, go back later and touch them up in ink. Computer-drawn graphs are fine as long as they comply with the remaining guidelines.

2. Scale your axes to take the best possible advantage of the graph paper. That is, draw as large a graph as possible, but the divisions of the graph paper should correspond to some nice interval like 1, 2, or 5 (times some power of 10). If you have to make the graph smaller to get a nice interval, make it smaller, but check that you've picked the nice interval that gives you the largest graph. Making the graph large will display your data in as much detail as possible. When using log-log or semi-log paper, choose paper with the number of cycles that gives the largest possible graph.

3. The lower left-hand corner need not be the point $(0, 0)$. Choose the range of values for each axis to be just wide enough to display all the data you want. If $(0, 0)$ does not appear on the graph, it's customary (but not necessary) to mark the break in the axis with two wavy lines ($\approx$).

4. Mark the scale of each axis (the number of units corresponding to each division) for the entire length of the axis.
5. Label both axes, identifying the quantity being plotted on each axis *and* the units being used.
6. Give each graph a title or provide a figure caption. The title should summarize the information contained in the axes and also gives any additional information needed to distinguish this graph from other graphs in the report.
7. Give each graph a number (e.g., "Figure 2"), which you can use in the body of the report to refer quickly to the graph.
8. If you calculate the slope and intercept of the graph from two points (rather than using linear regression), indicate the two points you used on the graph. Draw the line through the two points, label it "Best-fit line" (or something similar), and give its slope and intercept on the graph in some large clear space.

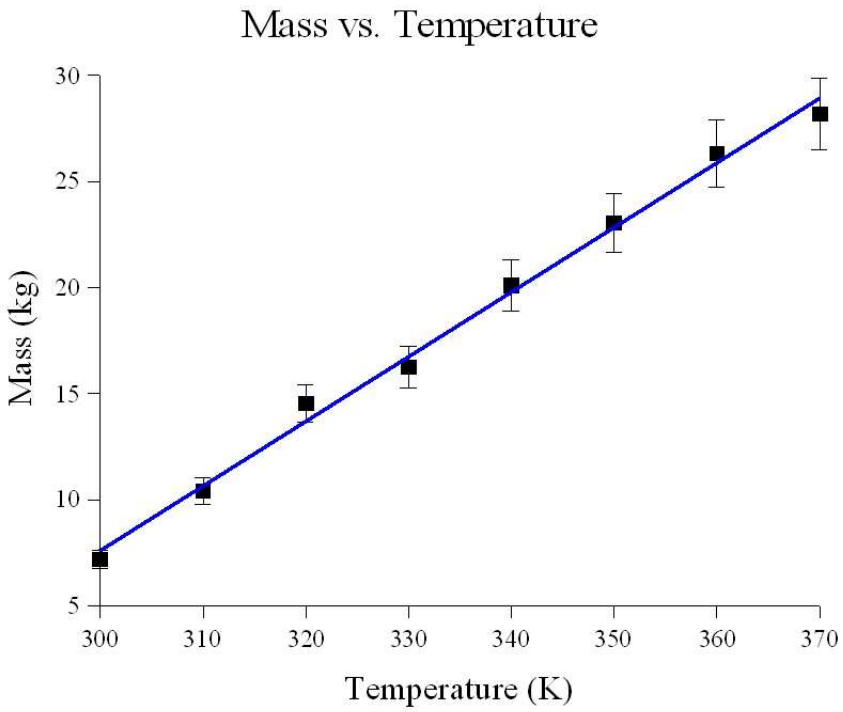

Figure B.2. Mass vs. Temperature - this is a sample graph illustrating all the features of a "high-level" graph. The solid line represents the best fit to the sample data.

Graphing Checklist

- Axes scaled correctly with divisions equal to "nice" intervals (1,2,5 or 10);

- Graph drawn to as large a scale as possible;
- Scales on axes labeled for entire length;
- Axes labeled, including units;
- Graph titled and numbered; and
- Points used to calculate slope and intercept clearly marked, if that method is used.

Appendix C

Linear Regression Analysis

C.1 Introduction

The purpose of this appendix is to introduce you to a method for finding the slope and the intercept of the straight line that "best" represents your data in the presence of the inevitable experimental uncertainty of your measurements. This method is known as "linear least squares" or "linear regression." Because the entire procedure is somewhat involved, we are breaking up the background material into two parts. We do not expect you to be able to reproduce this derivation, but we feel that it is important for you to know, at least in outline form, how the method of least-squares works.

We assume that we have accumulated a set of N "observed" values $y_1^{\text{obs}}, y_2^{\text{obs}}, ..., y_N^{\text{obs}}$, each with an associated value of the independent variable $x_1, x_2, ..., x_N$.

To develop a procedure for determining the line that "best" fits a set of experimental data points, we first must agree on a quantitative criterion for the "best" fit. Several criteria are possible, but the one most commonly used is called a "least-squares" best fit. This expression means that the square of some quantity will be minimized by the choice of slope and intercept of the line. The specific quantity that is minimized is the sum of the squares of all the discrepancies between the observed data points and the values calculated from the slope and intercept. That is, suppose that the "true" equation describing your data has the form $y^{\text{calc}} = mx + b$, where as usual m is the slope and b is the y-intercept of the straight line. The least-squares best-fit line is the line with the slope and intercept that minimizes the quantity

$$S = \sum_{i=1}^{N} (y_i^{\text{obs}} - y_i^{\text{calc}})^2 = \sum_{i=1}^{N} (y_i^{\text{obs}} - mx_i - b)^2. \tag{C.1}$$

At the moment, you don't know the values of m and b, but those values can be found using

the procedure below.

For each of your N values of the independent variable x_i you have a measured value y_i of the dependent variable. If you knew the values of m and b, you could also predict a value for $y(x_i)$ given the measured value of x_i. In general, your predicted and observed values of y_i for a given x_i would not agree. As stated above, *the least-squares procedure minimizes the sum of the squares of the amounts by which the observed and calculated values disagree with each other.*

But, you may argue, you still don't know the values of the slope and intercept to make these calculations. It turns out that m and b can be calculated from your experimental results. Here's how to do it: call the sum of the squared differences S, and write an expression for S in terms of the unknown coefficients m and b, and your collection of N measured data points x_i and y_i as we did in the second part of Eq. C.1.

S may be minimized by the usual method of setting the derivative to zero, but since S depends on two unknown quantities, m and b, we must calculate two derivatives, called *partial* derivatives. We calculate each partial derivative by treating the other unknown as a constant for the purposes of that derivative.

Using just y_i for y_i^{obs}, we write the two partial derivatives for S as follows:

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{N}(-2)(y_i - mx_i - b) \tag{C.2}$$

and

$$\frac{\partial S}{\partial m} = \sum_{i=1}^{N}(-2x_i)(y_i - mx_i - b). \tag{C.3}$$

Setting these derivatives equal to zero, for a minimum, dividing through by the factor of 2, and rearranging terms for purely aesthetic reasons yields

$$\sum_{i=1}^{N}(b + mx_i) = \sum_{i=1}^{N} y_i \tag{C.4}$$

and

$$\sum_{i=1}^{N}(bx_i + mx_i^2) = \sum_{i=1}^{N} x_i y_i. \tag{C.5}$$

The constants m and b, although unknown, can be factored out of the summations. This factorization leaves behind a slightly mysterious-looking term int he first equation, since

$$\sum_{i=1}^{N} b = b\sum_{i=1}^{N} 1 = bN \tag{C.6}$$

since the sum of 1 over N observations is just N.

We may now write Eqs. C.4 and C.5 as

$$Nb + m\sum x_i = \sum y_i \tag{C.7}$$

$$b\sum x_i + m\sum x_i^2 = \sum x_i y_i \tag{C.8}$$

where we have left off the limits of the sums for the sake of typographical simplicity.

The sums in Eqs. C.7 and C.8 may be calculated from your data and should be treated as known quantities. Hence you have two equations in two unknowns, m and b, and the procedure for finding them is straightforward. Solving the equations above for m and b gives

$$m = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{N(\sum x_i^2) - (\sum x_i)^2} \tag{C.9}$$

and

$$b = \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i)\sum(x_i y_i)}{N(\sum x_i^2) - (\sum x_i)^2}. \tag{C.10}$$

Equations C.9 and C.10 give you an expression for the slope m and intercept b of the "best-fit" straight line for your data, in terms of your measurements. Remember here that best-fit means the line that minimizes the squares of the differences between the observed y values and the calculated y values. (If you have a reasonably standard scientific calculator, it probably has a "canned" routine for performing this kind of fit.)

EXERCISE I

Performing a linear regression "by hand" (with the assistance of a calculator).

Our experience suggests that people find the derivation above somewhat bewildering, apparently as a result of all the summation signs. To make the process of linear regression seem more familiar, in this exercise you will carry out one linear regression calculation (with a small number of points) by hand, following the worksheet below. You will also check that the calculated slope and intercept do indeed give a smaller value of the squared differences than other, similar values. We have filled in the first row and a few other spaces so that you can check your work.

measurement	X	X^2	Y	XY
1	0.6	0.36	0.45	0.27
2	1.5		1.10	
3	2.3		1.35	
4	3.0		1.65	
totals	7.4			

$$\text{slope} = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2} \tag{C.11}$$

$$\text{intercept} = \frac{(\sum X^2)(\sum Y) - (\sum X)(\sum XY)}{N(\sum X^2) - (\sum X)^2}. \tag{C.12}$$

After carrying out the calculations above, plot the four data points on a sheet of graph paper. Then add your best-fit line, and see how well the line and the observed data seem to agree. Then compare the data and your best-fit line to two other similar lines: $y = 0.5x + 0.25$ and $y = 0.45x + 0.3$. (That is, plot those lines on the graph as well.)

Now compare the squared differences using your calculated slope and intercept to squared differences using the other two trial lines above. In the tables below, Y_{obs} is the observed value of Y from the first table, and is filled in for you. Y_{calc} is the value of Y you calculate from the trial values of m and b and the value of X.

Trial 1: Use $m = 0.5$, $b = 0.25$.

X	Y_{obs}	Y_{calc}	$(Y_{\text{obs}} - Y_{\text{calc}})^2$
0.6	0.45	0.55	0.01
1.5	1.10		
2.3	1.35		
3.0	1.65		
			0.0325 (total)

Trial 2: Use $m = 0.45$, $b = 0.3$.

Trial 3: Use your values of m and b from the least-squares procedure.

C.2 Uncertainty in the Slope and the Intercept

The slope m and intercept b of the "best-fit"' line, calculated above, are of course based on experimental data. Like the data, they must be uncertain to some degree or other. In practice, we can calculated the standard deviations to be associated with the slope and intercept values using the same data we used to find the slope and intercept. However, the calculations and their derivations are beyond the scope of this course. The computer programs Excel and Graphical Analysis available on the computers in the lab can do the

"linear regression" calculations for you. But what can such a program tell you about the uncertainty in your slope resulting from the data?

Ideally, you would like a computer program to accept information about the experimental uncertainties in your measured values and to combine them to give you an uncertainty in the results for the slope and intercept. You already know how to do a calculation by hand for the uncertainty in, for example, the product of two measured values. (For a review of the method, refer to Appendix F.) For the collection of data points that are presented on a graph, however, the overall uncertainties in the slope and intercept must result from some composite consideration of the uncertainties in all of the data a t once. Some graphing programs (such as Excel) do allow for such a calculation. But what does it mean? Let us first consider a graph containing only the raw data points for which no "uncertainty bars" are included. (You can refer to Appendix B, Graphical Presentation of Data, for a discussion of uncertainty bars.)

You can get a feel for the effect of the uncertainties in the slope and intercept due solely to the scatter in the data by identifying two "worst cases" that are still consistent with the data. One case is the line that is systematically a little too low at the left-hand side of the graph and too high and the right-hand side, but fits the data well near the middle. This line combines the largest acceptable slope with the smallest acceptable intercept; that is, its slope is $m + \Delta m$ and its intercept is $b - \Delta b$, where Δm and Δb are the uncertainties associated with the slope and intercept, respectively. The other worst case combines the smallest acceptable slope with the largest acceptable intercept; this line is a little too high at the left end, a little too low at the right end, and a fairly good fit in the middle. Exercise II gives an example that allows you to visualize the extremes for a fit line.

EXERCISE 2

Assume that, for the data above, the uncertainty in the slope is $\Delta m = 0.064$ and the uncertainty in the intercept is $\Delta b = 0.13$. Plot a second graph showing the original data, the best-fit line, and the two "worst acceptable lines" determined from the uncertainties given above. Notice that the two worst acceptable lines seem to be heavily influenced by the ends of the graph.

C.3 The Correlation Coefficient and Other Loose Ends

Since now you have a nice numerical procedure for getting the "best" line, you may wonder if you still need to graph your data. Yes. Although the least-squares method will calculate the slope and the intercept of your line and the uncertainties in those quantities, it will not tell you if you have a bad data point. Nor can it decide if your data are close enough to being linear to justify fitting a straight line to them. You can only make these decisions

intelligently by plotting your data and examining them, particularly in cases where you don't have a good theory to go on.

Many calculators with built-in linear fitting routines include a calculation of a quantity called the correlation coefficient, usually called r. The correlation coefficient tells you how well "correlated" your data points are. A perfect fit (no experimental error) will have a correlation coefficient of 1, indicating perfect agreement between the observed and calculated values of y and suggesting that x and y are in fact related through $y = mx + b$. On the other hand, if x and y are completely unrelated to each other, $r = 0$. Real experimental data points always include some experimental error, so real correlation coefficients always have absolute magnitudes less than 1. If the correlation coefficient for your data is close to 1, you are justified in concluding that x and y are really related in some way. The correlation coefficient is therefore most useful when you're trying to decide if your data are good enough for you to claim that a variable y does in fact depend on another variable x.

The correlation coefficient is not very useful, however, when you're pretty sure (for, say, theoretical reasons) that y is determined by x. In that case, you're *assuming* that the correlation is valid, and you want to know the slope and the intercept of the line. The correlation coefficient will tell you in a general way how good your data are, but it doesn't tell you anything about the experimental uncertainty in the calculated values of the slope and the intercept. The correlation coefficient is also not very good for deciding if a linear fit is good enough, or if any of your measurements are suspect. It's surprisingly insensitive to curvature in the data, or the presence of a single dubious data point. For that reason we do not report the correlation coefficient with the results of the fit.

Appendix D

Power-Law Curve Fitting

Although many processes in nature are linear, many others are not. Power-law dependencies, of the form

$$y = kx^n \tag{D.1}$$

are particularly common. You have in fact already encountered one such relation in the simple pendulum experiment, the relation between the length of a pendulum and its period. In that experiment, you could use dimensional analysis to argue that the value of the exponent should be 0.5, but often you're not so lucky. And you don't know the value of the multiplying factor k either. What do you do?

One strategy, assuming you do know the exponent, amounts to organized trial and error to find k. You could, for example, try different values of k, calculate a $y(x)$ curve for each value, and see which value best matches your experimental data. This approach is clearly pretty tedious if you have only a calculator. And if you start with the wrong value of n, your value of k isn't going to be worth much. Sometimes you don't even have a value for n.

All is not lost. Suppose you take the logarithm (with respect to any base) of both sides of Eq. D.1. The result is

$$\log y = \log k + n \log x. \tag{D.2}$$

Now let $\log x = u$, and $\log y = v$. With this substitution and a slight rearrangement, Eq. D.2 becomes

$$v = nu + \log k. \tag{D.3}$$

Now you have the equation of a straight line. That is, if you graph $\log y$ vs. $\log x$, you should wind up with a straight line. Furthermore, the slope of this line is n, the value of the exponent in Eq. D.1. You find the value of n by calculating the slope in the usual way.

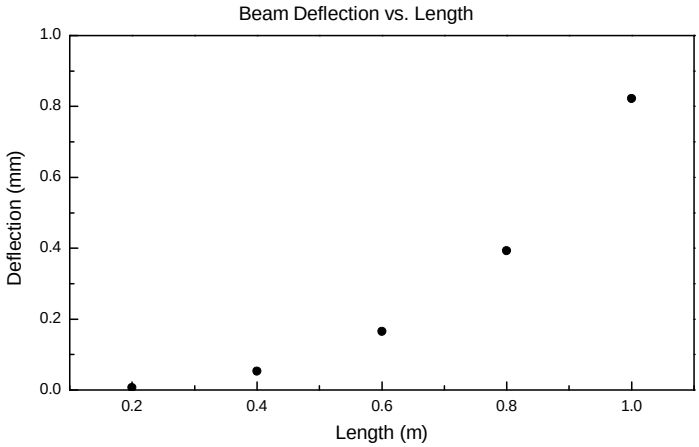

Figure D.1. Linear plot of beam deflection as a function of beam length.

Length (m)	Deflection (mm)
0.2	0.007
0.4	0.053
0.6	0.165
0.8	0.393
1.0	0.822

Table D.1. Beam deflection data.

That is,

$$n = \frac{\Delta v}{\Delta u} = \frac{v_2 - v_1}{u_2 - u_1} = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1}. \tag{D.4}$$

As an example, Fig. D.1 below shows data for an experiment studying the deflection of a loaded beam as a function of the length of the beam. The data are listed in Table D.1. A linear plot of the data is clearly not a straight line, but the curve might be a power law because it passes through the point $(0, 0)$. To test this hypothesis, plot $\log D$ vs. $\log L$; sure enough, the new graph, shown in Fig. D.2, looks pretty straight.

The log-log graph passes through the points $(-0.824, -2.538)$ and $(0.0, -0.084)$. These points are the logarithms of the points $(0.15, 0.0029)$ and $(1.0, 0.825)$. The slope of this line is therefore

$$\text{slope} = \frac{\log(0.825) - \log(0.0029)}{\log(1.0) - \log(0.15)} = \frac{-0.084 - (-2.538)}{0.0 - (-0.824)} = 2.98. \tag{D.5}$$

Calculating the logarithms if you have more than five or ten data points, even with a calculator, gets tedious after a while. To reduce this tedium, special graph paper called

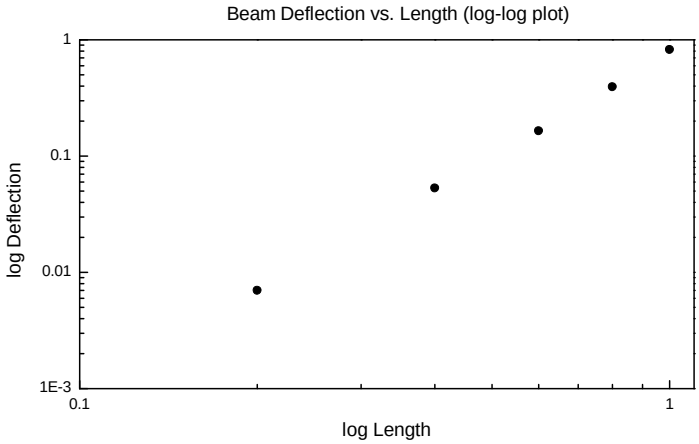

Figure D.2. Log-log plot of beam deflection as a function of beam length. The axes are labeled as they would appear on log-log graph paper.

"log-log" paper was invented. A sample is given at the end of this appendix. The logarithms of numbers are spaced uniformly along both the horizontal and vertical axes, as you can confirm by comparing the spacing of 2 and 4 and the spacing of 4 and 8 (or 8 and 16, and son). In effect, the graph paper calculates the logarithms for you.

Log-log paper is most useful when you suspect your data has a power-law dependence and you want to test your suspicion. Sometimes your suspicion is based on a theoretical prediction, sometimes on a previous linear plot. Figure D.3, below, is a typical linear graph that could be a power law with a **negative** exponent. (What happens as the independent variable goes to zero?)

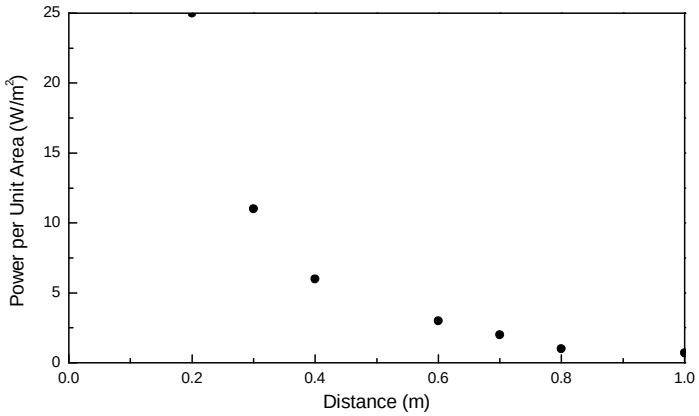

Figure D.3. Possible power-law behavior with a negative exponent.

For either positive or negative exponents, your next step is to plot your data on log-log paper. If the graph on log-log paper is a pretty good straight line, within your experimental

uncertainty, you can conclude that your data does indeed have a power-law dependence. You can find the constant k in Eq. D.1 by extrapolating the line back to $x = 1$, the equivalent of $\log x = 0$, and reading the value off the vertical axis. You still have to find the slope by calculating $\Delta(\log y)/\Delta(\log x)$, as shown above.

One further variation: some data may be described by a power law added to a constant term. For example,

$$y = kx^n + B, \tag{D.6}$$

where B is independent of x. How could you apply a log-log plot to test this hypothesis? Hint: we can estimate B by looking at what happens for small x (if n is positive). What would you do if n were negative?

Appendix E

Guidelines for Formal Laboratory Reports

The formal lab report should be a complete presentation of your work on the experiment. It should be written for someone who has a physics background equivalent to Physics 117, *but who does not know anything about the experiment and the measurements you carried out*. There are three principal components to every formal report:

- *Format*: the organization and presentation of the report.
- *Composition*: the style in which the report is written.
- *Content*: the subject matter of the report.

All three are essential for writing a complete and self-consistent report. The purpose of the reports is to test both your analytical skills and your writing skills at communicating physical concepts. It must be emphasized: formal reports are short papers, not questionnaires or "fill in the blank". The reports will be graded with same importance as papers in other course. All components of the report will critiqued in the grading.

E.1 Format

The report should be formatted in a way that clearly presents all the relevant information to the reader: text, equations, figures, etc. Some standard report formatting include:

- Typeset using a word-processor.

- Lines double spaced.
- 12 point Times New Roman font.
- 1" left and right margins.
- Text is full justification.
- Reports are no more than 6 pages long.

The organization of the report is crucial. The reader anticipates a particular order for the report to be presented. Deviation from that order will mislead or confuse the reader. The report should be organized as follows:

1. **Title:** The title should be a simple descriptive phrase, centered at the top of the first page of the report. Also include your name, the date, the lab section and the name(s) of your partner(s).
2. **Introduction:** The introduction is a short, single paragraph statement of the experiment. What is the purpose, the *main* goal, of experiment and why is the experiment a worthwhile means of exploring a particular physical concept?
3. **Theoretical background:** The theoretical background should state what the underlying physics of the experiment is. What the theory predicts, what assumptions have been made, and how the experiment relates to the theory of the physics being studied. Terminology specific to the experiment should be defined. Often, the theory can be best expressed analytically in the form of an equation. Define the quantities to be determined and how they are related to the directly measured quantities.
4. **Experimental technique:** The experiment technique should be a detailed *narrative* of the experimental procedure. *What* was measured and *how* was it measured? Include a simple diagram of the apparatus whenever possible. Indicate the primary sources of measurement uncertainty. Give numerical estimates of uncertainties associated with each directly measured quantity.
5. **Data, analysis, and results:** Display the data in one or more appropriate forms (tables, graphs, etc). Discuss how the final results are obtained. Give estimates of the uncertainty of the results based upon measurements uncertainties. Be sure to include some discussion of experimental uncertainties and how those uncertainties affect the evaluation of your results.
6. **Discussion of results and Conclusion:** The conclusion should reflect your overall understanding of the experiment, i.e., what have you learned about the particular subject of physics studied in the experiment? It should consist of a logical sequence of statements substantiated by the evidence presented in the report. Was the goal of the experiment accomplished? Were the experimental results *consistent* with theoretical expectations? That is, do they agree within the range of uncertainty? What are the

implications of your results? It is good practice to restate any numerical results in the conclusion for easy review by the reader.

Other forms of information require specific formatting:

- **Equations:** Equations should be centered on separate lines from the text of the report. Each equation should be numbered, preferably along the right margin, for easy reference. An equation is often followed by a sentence that defines the variables in the equation. Equations are especially useful when stating the theoretical background of the report. Do not include long derivations of equations in the report. Instead, simply reference which equations are used in the derivation and give final result. Keep derivations in your notebooks.
- **Figures - Diagrams:** diagrams of experimental setup should be simple yet illustrative of the experimental setup and apparatus. The relevant parts should be labeled and the relevant measured quantities indicated. Each diagram should have a figure number (Figure 1:) and caption below the diagram. The caption should be a concise description of the figure and any important parts. Use drafting tools like rules, protractors or applications like MS Paint to draw diagrams. Do not draw free hand diagrams.
- **Figures - Graphs:** The title of a graph should clearly indicate which two quantities are plotted. The title convention for a graph is Y (vertical axis) vs. X (horizontal axis). The axes should be labeled and include units. Graphs use the same convention of numbering and captions as diagrams (Figure 1:). The caption should be a brief description of the graph and the quantities plotted. Adjust scale of axes so data points fill the whole graph. Empty space is a waste. For more details refer to Appendix B, Graphical Presentation of Data.
- **Tables:** Data tables should be organized in columns. The head of each column should be labeled and include units. If the quantities in the column all have the same uncertainty, then the uncertainty can be indicated at the head of the column as well; for example, Time ($\pm$0.001 sec). Each table should have a descriptive title, starting with a number (Table 1:) for easy reference. Do not split tables across pages of the report. Do not include long tables like the data tables from motion sensor. Keep those tables in a spreadsheet.

E.2 Composition

A formal lab report is paper, similar to other papers written in other courses, and should follow the accepted conventions of composition. The report should be written in narrative style. Correct spelling, grammar, punctuation and syntax are essential.

Always make use of resources when writing: dictionary, thesausus, handbook. Probably the best handbook for college writing is Strunk and White *The Elements of Style*, 4th ed. It is particularly good in its no nonsense approach to writing, and it has been a standard handbook for collegiate writing for almost 100 years, and it is the standard handbook for this course. Amherst College Writing Center also has links on its webpage to several online resources.

The style of scientific writing is definitive, concrete and fact-based. It is not poetic, literary, sarcastic or opinionated. Specific styles of writing the *should not* be used in your report:

- Editorial - expressions of opinion or commentary. The language should be simple and substantive; based on the evidence presented in the report. It should not be an expression of how the writer feels about the experiment.
- Bombastic - inflated or grandiose language. Reports are not exercises in creative writing. The purpose of the report is to educate the reader *not* impress the reader. Keep the language simple.
- Verbiage - excessively wordy but conveying little or no information. Do not ramble.
- Abstract - void of concrete, real world meaning. Use physical terminology and use it correctly.
- Circumlocution - excessive use of words to explain a concept or idea. Do not over explain the experiment from first principles. Be specific to the subject matter.

Standard conventions include:

- **Write in narrative prose, not outline form.** This is especially true when writing sections of the report like the experimental technique. Do not recite the procedure outlined in the lab manual.
- **Use simple, complete sentences.** Sentences are expressions of one complete thought, fact or idea. The simpler the sentence, the better. Avoid excessive use of qualifiers, modifier and subordinate clauses and phrases. Get to the point and stay on topic.
- **Write in paragraph style.** The paragraph is the building block of the report. Each paragraph should address one topic of the report. Indent!
- **Use correct terminology, spelling and grammar.** There are some words in everyday language that have specific meaning in the context of physics. Make sure the terminology is consistent with the subject matter of the report. Always use the standard spelling of words.

- **Write formally; do not use slang or colloquialisms.** Do not write in a casual manner; for example, the word "plug", as in, "I plugged the numbers into the equation.." This is a sloppy, lazy style of writing. Formal reports should be written in a formal style.

E.3 Content

The content of the report addresses the subject matter; the principles, ideas and concepts the report is about. Since we believe physics is a logical, self-consistent science, the content of the report should be logical and self-consistent as well.

- **Clarity**: the underlying principles are clearly articulated, all relevant terminology is defined. You should be specific in the language used. Avoid vague or ambiguous statements.
- **Completeness**: all elements of the report are present. Missing or omitted content will mislead or confuse the reader.
- **Conciseness**: specific and to the point. The writer should avoid redundant, irrelevant or circuitous statements. Stay on topic.
- **Consistency**: all elements of the report direct the reader to a single, logical conclusion. Avoid illogical, erroneous, unsubstantiated, specious, irrelevant statements and contradictions.
- **Continuity**: all elements of the report follow a logical order. The discussion is constructed in a sequential manner. Avoid incoherent, disorganized statements.

E.4 Questions and Exercises

In some experiments specific questions and exercises will be asked. The purpose of the questions and exercises is to motivate the discussion. Questions and exercises should be answered within the body of the report and always from a *physics* point of view. The report is incomplete without answers to questions and exercises.

E.5 Some general writing guidelines

- Do not assume too much about the reader's knowledge of the experiment. It's your responsibility to explain the subject matter to the reader. Assuming the reader already

understands the subject and not providing a complete explanation makes the report seem disjointed.

- Proofread! Spelling and grammatical errors are easy to fix; otherwise the report appears sloppy. Make use of resources: dictionary, thesaurus, *Strunk and White*, etc.

- Check the Units, Significant Figures and Uncertainties! Values without units are meaningless. Make sure all the values have the correct units. Checking the unit of a value is also a good cross check of a calculation. For example, if a calculation is solving for a velocity (m/sec) but the solution has units in kilograms, then there may be an error in the calculation.

- Do not mix together discussion of theory, procedure and analysis. Use subheadings; for example, Theory, Analysis, Conclusion, to keep the report organized and the reader's attention focused.

- Do not over explain the experimental setup. A well drawn diagram of the setup is better than a lot of prose. Keep the topic of the report on the *physics*.

- Do not over explain the use of instruments like computers, calculators or software like Excel©. Computers and softwares are only tools. Keep to the *physics*.

- Always present a result in the form $x_{meas} = x_{best} \pm \Delta x$. It is nonsense to present the best value separate from the uncertainty.

- Do not wrap text around figures, tables or equations.

- Do not include long tables of data in report; especially data from motion sensors. Long tables are boring to the reader and the data are better presented in a graph.

- Do not include summary Output Page from Regression Analysis in report. Keep Output Page in your notebook. Only extract the necessary values for you report.

- Do not include long calculations and algebra derivations. Reference the equation that is evaluated and state result. Only give derivations if it is asked for in a specific exercise. Keep a record of any detail calculations in your lab notebook.

- Use drafting tools like rules, protractors, compasses or or applications like MS Paint to draw diagrams. No free-hand diagrams.

- DO NOT DOWNLOAD FIGURES FROM INTERNET! Produce your own figures.

- Ask for help. Don't be afraid to ask questions if you are unclear on something. There is no need to guess what we want in the report.

Appendix F

Uncertainty Analysis

An intrinsic feature of every measurement is the uncertainty associated with the result of that measurement. No measurement is ever exact. Being able to determine and assess measurement uncertainties intelligently is an important skill in any type of scientific work. The measurement (or experimental) uncertainty should be considered an *essential* part of every measurement.

Why make such a fuss over measurement uncertainties? Indeed, in many cases the uncertainties are so small that, for some purposes, we needn't worry about them. On the other hand, there are many situations in which small changes might be very significant. A clear statement of measurement uncertainties helps us assess deviations from expected results. For example, suppose that two scientists report measurements of the speed of light (in vacuum). Scientist Curie reports 2.99×10^8 m/s. Scientist Wu reports 2.98×10^8 m/s. There are several possible conclusions we could draw from these reported results:

1. These scientists have discovered that the speed of light is not a universal constant.
2. Curie's result is better because it agrees with the "accepted" value for the speed of light.
3. Wu's result is worse because it disagrees with the accepted value for the speed of light.
4. Wu made a mistake in measuring the speed of light.

Without knowing the uncertainties in these measurements, however, it turns out that we cannot assess the results at all!

F.1 Expressing Experimental Uncertainties

Suppose that we have measured the distance between two points on a piece of paper. There are two common ways of expressing the uncertainty associated with this measurement: absolute uncertainty and relative uncertainty. In both ways the measured quantity is expressed in the form:

$$x_{measured} = x_{best} \pm \delta x \tag{F.1}$$

Here x_{best} is the best measured value, usually from an average of a set of measurements, and δx is the uncertainty in the best measured value. The measurement is always a *range* of values, not just the best value.

F.1.1 Absolute Uncertainty

We might express the result of the measurement as

$$5.1 \text{ cm} \pm 0.1 \text{ cm}. \tag{F.2}$$

By this we mean that the result (usually an average result) of the set of measurements is 5.1 cm, but given the conditions under which the measurements were made, the fuzziness of the points, and the refinement of our distance measuring equipment, it is our best judgment that the "actual" distance might lie between 5.0 cm and 5.2 cm.

Incidentally, an alternative (shorthand) way of expressing this uncertainty looks like this:

$$5.1(1) \text{ cm} \tag{F.3}$$

where the number in parentheses represents the uncertainty in the last digit. Feel free to use this form in your lab work.

F.1.2 Relative (or Percent) Uncertainty

The relative uncertainty is defined:

$$f_x = \frac{\delta x}{|x_{best}|} \tag{F.4}$$

We might express the same measurement result as

$$x_{measured} = x_{best} \pm f_x \tag{F.5}$$

For example:

$$5.1 \text{ cm} \pm 2\%. \tag{F.6}$$

Here the uncertainty is expressed as a percentage of the measured value. Both means of expressing uncertainties are in common use and, of course, express the same uncertainty.

F.1.3 An aside on significant figures

The number of significant figures quoted for a given result should be consistent with the uncertainty in the measurement. In the example, it would be inappropriate to quote the results as 5 cm$\pm$0.1 cm (too few significant figures in the result) or as 5.132 cm$\pm$0.1 cm (too many significant figures in the result). Some scientists prefer to give the best estimate of the next significant figure after the one limited by the uncertainty; for example, 5.13 cm$\pm$0.1 cm. The uncertainties, since they are estimates, are usually quoted with only one significant figure; in some cases, *e.g.*, for very high precision measurements, the uncertainties may be quoted with two significant figures.

F.1.4 Rules for significant figures

1. All nonzero digits are significant. Ex.: 1-9.
2. Zeros between nonzero digits are significant. Ex.: 230504 (6 significant figures).
3. Leading zeros to left of nonzero digit are not significant. Such zeros only indicate position of decimal point. Ex.: 0.002 (1 significant digit).
4. Trailing zeros to right of decimal point are significant. Ex.: 0.0340 (3 significant digits).
5. Trailing zeros to the left of the decimal point may or may not be significant. Ex.: 50,600 (3,4 or 5 significant figures).
6. When adding or subtracting numbers, the final answer is round off to the decimal place equal to the number with the fewest decimals.
7. When multiplying or dividing numbers, the final answer is round off the same number of significant figures equal to the number with the fewest significant figures.

F.2 Determining Experimental Uncertainties

There are several methods for determining experimental uncertainties. Here we mention three methods, which can be used easily in most of the laboratory measurements in this course.

F.2.1 Estimate Technique

In this method, we estimate the precision with which we can measure the quantity of interest, based on an examination of the measurement equipment (scales, balances, meters,

etc.) being used and the quantity being measured (which may be "fuzzy," changing in time, etc.). For example, if we were using a scale with 0.1 cm marks to measure the distance between two points on a piece of paper, we might estimate the uncertainty in the measured distance to be about ± 0.05 cm, that is, we could easily estimate the distance to within $\pm\frac{1}{2}$ of a scale marking.

F.2.2 Sensitivity Estimate

Some measurements are best described as comparison or "null" measurements, in which we balance one or more unknowns against a known quantity. For example, in the Wheatstone bridge experiment, we will determine an unknown resistance in terms of a known precision resistance by setting a certain potential difference in the circuit to zero. We can estimate the uncertainty in the resulting resistance by slightly varying the precision resistor to see what range of resistance values leads to a "balanced" bridge within our ability to check for zero potential difference.

F.2.3 Repeated Measurement (Statistical) Technique

If a measurement is repeated in independent and unbiased ways, the results of the measurements will be slightly different each time. A statistical analysis of these results then, it is generally agreed, gives the "best" value of the measured quantity and the "best" estimate of the uncertainty to be associated with that result. The usual method of determining the best value for the result is to compute the "mean value" of the results: If $x_1, x_2, ..., x_N$ are the N results of the measurement of the quantity x, then the mean value of x, usually denoted $\overline{x}$, is defined as

$$\overline{x} \equiv \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{F.7}$$

The uncertainty in the result is usually expressed as the "root-mean-squared deviation" (also called the "standard deviation") usually denoted as δx (read "delta x"). [Note that here δx does not mean the change in x, but rather is a measure of the spread in x values in the set of measurements.] Formally, the standard deviation is computed as

$$\delta x = \sqrt{\frac{(x_1 - \overline{x})^2 + \cdots + (x_N - \overline{x})^2}{N-1}} \tag{F.8}$$

Although determining the standard deviation may be tedious for a large array of data, it is generally accepted as the "best" estimate of the measurement uncertainty.

N.B.: In general, we cannot expect exact agreement among the various methods of determining experimental uncertainties. As a rule of thumb, we usually expect the different methods of determining the uncertainty to agree within a factor of two or three.

EXAMPLE

Suppose that five independent observers measure the distance between two rather fuzzy marks on a piece of paper and obtain the following results:

$d_1 = 5.05$ cm
$d_2 = 5.10$ cm
$d_3 = 5.15$ cm
$d_4 = 5.20$ cm
$d_5 = 5.10$ cm

If the observers were using a scale with 0.1 cm markings, method 1) would suggest an uncertainty estimate of about ± 0.05 cm. Method 3) yields a mean value $d = 5.12$ cm and for the standard deviation 0.057 cm ~ 0.06 cm. We see that in this case we have reasonable agreement between the two methods of determining the uncertainties. We should quote the result of this measurement as $5.12 \text{ cm} \pm 0.06$ cm or $5.12 \text{ cm} \pm 1\%$.

F.3 Propagation of Uncertainties

In most measurements some calculation is necessary to link the measured quantities to the desired result. The question then naturally arises: How do the uncertainties in the measured quantities affect (propagate to) the results? In other words, how do we estimate the uncertainty in the desired result from the uncertainties in the measured quantities?

F.3.1 "High-Low" Method

One way to do this is to carry through the calculation using the extreme values of the measured quantities, for example 5.06 cm and 5.18 cm from the previous example, to find the range of result values. This method is straightforward but quickly becomes tedious if several variables are involved.

EXAMPLE

Suppose that you wish to determine a quantity, X, which is to be calculated indirectly using the measurements of a, b, and c, together with a theoretical expression: $X = \frac{ab}{c}$.

Suppose, further, that you have already determined that

$a = 23.5 \pm 0.2$ m
$b = 116.3 \pm 1.1$ N
$c = 8.05 \pm 0.03$ s

The "best" value of X is

$$X_{\text{best}} = \frac{23.5 \times 116.3}{8.05} = 339.509 \text{ N} \cdot \text{m/s} \tag{F.9}$$

(We'll clean up the significant figures later.)

But X could be about as large as what you get by using the maximum values of a and b and the minimum (why?) value of c:

$$X_{\text{high}} = \frac{23.7 \times 117.4}{8.02} = 346.930 \text{ N} \cdot \text{m/s}. \tag{F.10}$$

And similarly, we find

$$X_{\text{low}} = \frac{23.3 \times 115.2}{8.08} = 332.198 \text{ N} \cdot \text{m/s}. \tag{F.11}$$

Notice that X_{high} and X_{low} differ from X_{best} by about the same amount (7.3). Also note that it would be silly to give six significant figures for X. Common sense suggests reporting the value of X as, say, $X = 339.5 \pm 7.3$ N · m/s, or $X = 339 \pm 7$ N · m/s.

F.3.2 General Method

The general treatment of the propagation of uncertainties is given in detail in texts on the statistical analysis of experimental data. A particularly good reference at this level is Taylor [2]. Here we will develop a very simple, but general method for finding the effects of uncertainties.

Suppose we want to calculate some result R, which depends on the values of several measured quantities x, y, and z:

$$R = f(x, y, z) \tag{F.12}$$

Let us also suppose that we know the mean values and standard deviations for each of these quantities. Then the uncertainty in R due to the uncertainty in x, for example, is calculated from

$$\delta_x R = \left(\frac{\partial}{\partial x} f(\overline{x}, \overline{y}, \overline{z}) \right) \delta x \tag{F.13}$$

where the subscript on δ reminds us that we are calculating the effect due to x alone. Note that the partial derivative is evaluated with the mean values of the measured quantities. In a similar fashion, we may calculate the effects due to δy and δz.

N.B. By calculating each of these contributions to the uncertainty individually, we can find out which of the variables has the largest effect on the uncertainty of our final result. If we want to improve the experiment, we then know how to direct our efforts.

We now need to combine the individual contributions to get the overall uncertainty in the result. The usual argument is the following: If we assume that the variables are independent so that variations in one do not affect the variations in the others, then we argue that the net uncertainty is calculated as the square root of the sum of the squares of the individual contributions:

$$\delta R = \sqrt{(\delta_x R)^2 + (\delta_y R)^2 + (\delta_z R)^2} \tag{F.14}$$

The formal justification of this statement comes from the theory of statistical distributions and assumes that the distribution of successive measurement values is described by the so-called Gaussian (or equivalently, normal) distribution.

In rough terms, we can think of the fluctuations in the results as given by a kind of "motion" in a "space" of variables x, y, and z. If the motion is independent in the x, y, and z directions, then the net "speed" is given as the square root of the sum of the squares of the "velocity" components. In most cases, we simply assume that the fluctuations due to the various variables are independent and use Eq. F.14 to calculate the net effect of combining the contributions to the uncertainties.

Note that our general method applies no matter what the functional relationship between R and the various measured quantities. It is not restricted to additive and multiplicative relationships as are the usual simple rules for handling uncertainties.

In most cases, we do not need extremely precise values for the partial derivatives, and we may compute them numerically. For example,

$$\frac{\partial f}{\partial x} = \frac{f(\overline{x} + \delta x, \overline{y}, \overline{z}) - f(\overline{x}, \overline{y}, \overline{z})}{\delta x}. \tag{F.15}$$

Connection to the traditional simple rules

To see where the usual rules for combining uncertainties come from, let's look at a simple functional form:

$$R = x + y \tag{F.16}$$

Using our procedure developed above, we find that

$$\delta_x R = \delta x, \delta_y R = \delta y \tag{F.17}$$

and combining uncertainties yields

$$\delta R = \sqrt{(\delta x)^2 + (\delta y)^2} \tag{F.18}$$

The traditional rule for handling an additive relationship says that we should add the two (absolute) uncertainty contributions. We see that the traditional method overestimates the uncertainty to some extent.

Exercise: Work out the result for a multiplicative functional relationship $R = f(x, y) = xy$. Compare our method with the traditional method of "adding relative uncertainties."

Example

Suppose we have made some measurements of a mass m, a distance r, and a frequency f, with the following results for the means and standard deviations of the measured quantities:

$m = 150.2 \pm 0.1$
$r = 5.80 \pm 0.02$
$f = 52.3 \pm 0.4$

(Note that we have omitted the units and hence lose 5 points on our lab report.)

From these measured values we want to determine the "best value" and uncertainty for the following computed quantity:

$$F = mrf^2 \tag{F.19}$$

The "best value" is computed by simply using the best values of m, r, and f: $F = 2382875.2$ (we'll tidy up the number of significant figures later on).

Let's use our partial derivative method to find the uncertainty. First, let's determine the effect to to m:

$$\delta_m F = \left(\frac{\partial F}{\partial m}\right)\delta m = rf^2\delta m = 1586 \tag{F.20}$$

Next, we look at the effect of r:

$$\delta_r F = \left(\frac{\partial F}{\partial r}\right)\delta r = mf^2\delta r = 8217 \tag{F.21}$$

And finally, the effect of f is given by

$$\delta_f F = \left(\frac{\partial F}{\partial f}\right)\delta f = 2mrf\delta f = 36449 \tag{F.22}$$

We see immediately that the measurement of f has the largest effect on the uncertainty of F. If we wanted to decrease the uncertainty of our results, we ought to work hardest at decreasing the uncertainty in f.

Finally, let's combine the uncertainties using the "square root-of-the-sum-of-the-squares" method. From that computation we find that we ought to give F in the following form:

$$F = (2.383 \pm 0.037) \times 10^6 \tag{F.23}$$

or

$$F = (2.38 \pm 0.04) \times 10^6 \tag{F.24}$$

in the appropriate units. Note that we have adjusted the number of significant figures to conform to the stated uncertainty. As mentioned above, for most purposes, citing the uncertainty itself to one significant figure is adequate. For certain, high precision measurements, we might cite the uncertainty to two significant figures.

F.4 Assessing Uncertainties and Deviations from Expected Results

The primary reason for keeping track of measurement uncertainties is that the uncertainties tell us how much confidence we should have in the results of the measurements. If the results of our measurements are compared to the results expected on the basis of theoretical calculations or on the basis of previous experiments, we expect that, if no mistakes have been made, the results should agree with each other within the combined uncertainties. (Note that even a theoretical calculation may have an uncertainty associated with it because there may be uncertainties in some of the numerical quantities used in the calculation or various mathematical approximations may have been used in reaching the result.) There are several ways to assess whether our data support the theory we are trying to test.

F.4.1 Rule of Thumb

As a rule of thumb, if the measured results agree with the expected results within a factor of about two times the combined uncertainties, we usually can view the agreement as satisfactory. If the results disagree by more than about two times the combined uncertainties, something interesting is going on and further examination is necessary.

Example

Suppose a theorist from Harvard predicts that the value of X in the previous example should be 333 ± 1 N · m/s. Since our result (339 ± 7 N · m/s) overlaps the theoretical prediction within the combined uncertainties, we conclude that there is satisfactory agreement between the measured value and the predicted value given the experimental and theoretical uncertainties. However, suppose that we refine our measurement technique and get a new result 340.1 ± 0.1 N · m/s. Now the measured result and the theoretical result to not agree.

[Note that our new measured result is perfectly consistent with our previous result with its somewhat larger uncertainty.] We cannot tell which is right or which is wrong without further investigation and comparison.

F.4.2 χ^2 (Statistical) Technique

The χ^2 technique produces a number which tells you how well your data match the theory you are trying to test. Let's imagine that we have a theory which suggests that two measured variables x and y are related: $y = y(x)$. As a test of this theory let's further imagine that we do an experiment which generates x_i and $y_i \pm \sigma_i$. We are assuming here that the error in the x_i is negligibly small. The χ^2 for this data set is given by

$$\chi^2 = \sum_{i=1}^{N} \left\{ \frac{[y_i - y(x_i)]^2}{\sigma_i^2} \right\} \tag{F.25}$$

where N is the number of data points. The reduced χ^2, written χ^2_ν, is defined by

$$\chi^2_\nu \equiv \frac{\chi^2}{\nu} \tag{F.26}$$

where $\nu = N - n$ and n is the number of free parameters in the fit. (A straight-line fit has two free parameters, the slope and the intercept.)

Despite its appearance, χ^2_ν is fairly easy to interpret. To see this, let's consider a large set of data with only a small number of free parameters so that $\nu \approx N$. If the data fit well to the theory then we would expect, on average, that $[y_i - y(x_i)]^2 \approx \sigma_i^2$. Thus, we would also expect that $\chi^2 \approx N$ and therefore $\chi^2_\nu \approx 1$. If, on the other hand, our theory doesn't fit the data well, we would expect $[y_i - y(x_i)]^2 > \sigma_i^2$ on average and, therefore, $\chi^2_\nu > 1$. If we have overestimated our errors or chosen too many free parameters then $[y_i - y(x_i)]^2 < \sigma_i^2$ on average, and $\chi^2_\nu < 1$. A full analysis of this technique is given in Bevington [3]. In practice this is a difficult technique to apply with any rigor because estimating errors is so difficult. For the purposes of this course we will call any fit with $0.5 < \chi^2_\nu < 2$ a good fit.

F.5 The User's Guide to Uncertainties

The rules can be derived using the results of Sec. F.3.2.

F.5.1 Addition and Subtraction

For addition and subtraction one should combine the absolute uncertainties in the measured quantities. Typically, one calculates the final uncertainty by adding the uncertainties in quadrature, which means taking the square root of the sums of the squares. For example, the quadrature sum of the three uncertainties δx_1, δx_2, and δx_3 is

$$\delta x_{\text{total}} = \sqrt{(\delta x_1)^2 + (\delta x_2)^2 + (\delta x_3)^2}. \tag{F.27}$$

The following simple example shows how to propagate uncertainties for the case of a simple sum.

Example 1: Alice measures the lengths of the sides of a triangle, finding $s_1 = 2.9 \pm 0.2$ cm, $s_2 = 4.2 \pm 0.4$ cm, and $s_3 = 4.9 \pm 0.1$ cm. What is the perimeter of the triangle and Alice's uncertainty in its value?

Answer: The equation for the perimeter is $P = s_1 + s_2 + s_3 = 12.0$ cm. The final uncertainty in its value is found by summing the individual errors in quadrature:

$$\delta P = \sqrt{(\delta s_1)^2 + (\delta s_2)^2 + (\delta s_3)^2} = 0.5 \text{ cm}. \tag{F.28}$$

The value Alice should quote for the perimeter is therefore $P = 12.0 \pm 0.5$ cm.

F.5.2 Multiplication and Division

For multiplication and division, uncertainties propagate in a slightly different manner. One must first calculate the fractional uncertainty of a quantity. If some value q has an associated uncertainty δq, then

$$\text{fractional uncertainty} \equiv \frac{\delta q}{q} \tag{F.29}$$

Once we know the fractional uncertainties for each measured quantity in the product or quotient, we can add them in quadrature to get the fractional uncertainty of the result. To get the absolute uncertainty of the result, simply multiply the fractional uncertainty by the result.

Example 2: Bob wants to find the area of a triangle. He knows the length of the base $b = 4.2 \pm 0.2$ cm and the height $h = 5.8 \pm 0.1$ cm. What is the area of Bob's triangle?

Answer. The equation for the area is $A = \frac{1}{2}bh = 12.2$ cm^2. The final uncertainty in the result is found by summing the fractional errors in quadrature, and then multiplying by the result:

$$\delta A = A\sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta h}{h}\right)^2} = 0.6 \text{ cm}^2. \tag{F.30}$$

Bob should quote his total area as $A = 12.2 \pm 0.6$ cm^2.

F.5.3 Multiple Operations

For combinations of operations, the best approach is to break the problem up into pieces that can be solved by using the rules given above, and then combine the uncertainties of each of these pieces appropriately. The following example should make it clear how one can go about finding the final uncertainty in a more complicated problem.

Example 3. Cassandra wishes to know the speed of a cart traveling along a (level) air track. She measures the distance of two photogates from the end of the air track ($d_1 = 18.4 \pm 0.2$ cm and $d_2 = 160.1 \pm 0.3$ cm), and also the times at which the cart triggers each photogate ($t_1 = 0.53 \pm 0.01$ s and $t_2 = 1.88 \pm 0.02$ s. What is the speed of the cart and the uncertainty that Cassandra should quote?

Answer: The expression for the speed is, of course,

$$v = \frac{d_2 - d_1}{t_2 - t_1} \tag{F.31}$$

First, we compute the numerator and its uncertainty:

$$d_2 - d_1 = 141.7 \pm 0.4 \text{ cm} \tag{F.32}$$

where we applied the rules for addition and subtraction (add absolute uncertainties in quadrature). We now do a similar calculation for the denominator:

$$t_2 - t_1 = 1.35 \pm 0.02 \text{ s}. \tag{F.33}$$

Finally, we calculate v, using the rules for multiplication and division on the uncertainties in Eqs. F.32 and F.33 (add fractional uncertainties in quadrature):

$$v = \frac{141.7 \pm 0.4 \text{ cm}}{1.35 \pm 0.02 \text{ s}} = 105 \pm 2 \text{ cm/s}. \tag{F.34}$$

F.5.4 Simplified Uncertainty Rules

1. For a sum

   Add the absolute uncertainties in quadrature, i.e.

$$\text{If } A = B + C \text{ then } \delta A = \sqrt{(\delta B)^2 + (\delta C)^2} \tag{F.35}$$

2. For a difference

   Add the absolute uncertainties in quadrature, i.e.

$$\text{If } A = B - C \text{ then } \delta A = \sqrt{(\delta B)^2 + (\delta C)^2} \tag{F.36}$$

3. For a product

   Add the relative uncertainties in quadrature, i.e.

$$\text{If } A = B \times C \text{ then } \frac{\delta A}{A} = \sqrt{\left(\frac{\delta B}{B}\right)^2 + \left(\frac{\delta C}{C}\right)^2} \tag{F.37}$$

4. For a ratio

   Add the relative uncertainties in quadrature, i.e.

$$\text{If } A = \frac{B}{C} \quad \text{then } \frac{\delta A}{A} = \sqrt{\left(\frac{\delta B}{B}\right)^2 + \left(\frac{\delta C}{C}\right)^2} \tag{F.38}$$

5. For multiplication by a constant

   Multiple uncertainty by the constant, i.e.

$$\text{If } A = kB \quad \text{then } \delta A = k\delta B \tag{F.39}$$

6. For a square root

   Divide the relative uncertainty by 2.

$$\text{If } A = \sqrt{B} \quad \text{then } \frac{\delta A}{A} = \frac{1}{2}\frac{\delta B}{B} \tag{F.40}$$

7. For powers

   Multiple relative uncertainty by power, i.e.

$$\text{If } A = B^n \quad \text{then } \frac{\delta A}{A} = |n|\frac{\delta B}{B} \tag{F.41}$$

8. For functions

   Differentiate the function, i.e.

$$\text{If } A = A(x) \quad \text{then } \delta A = \left|\frac{dA}{dx}\right| \delta x \tag{F.42}$$

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[3] P. R. Bevington and D. K. Robinson. *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill, New York, NY, second edition, 1992.